Math 125 D - Winter 2017
Midterm Exam Number One
January 26, 2017

Name: ____________________ Student ID no. : ____________

Signature: ____________________ Section: ____________

1	15	
2	11	
3	6	
4	6	
5	10	
6	12	
Total	60	

- This exam consists of SIX problems on SIX pages, including this cover sheet.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic device are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- Draw a box around your final answer to each problem.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 80 minutes to complete the exam.

1. **[5 points per part]** Compute the indefinite integrals.

(a) $\int \frac{2-x^3}{x}\,dx$

$$= \int\left(\frac{2}{x} - x^2\right)dx = \boxed{2\ln|x| - \frac{1}{3}x^3 + C}$$

(b) $\int \left(\pi^x + \frac{3}{\sqrt{1-4x^2}}\right) dx$

$$= \frac{\pi^x}{\ln(\pi)} + \frac{1}{2}\int \frac{2\cdot 3}{\sqrt{1-4x^2}}\,dx = \frac{\pi^x}{\ln(\pi)} + \frac{3}{2}\int \frac{du}{\sqrt{1-u^2}}$$

$u = 2x$

$du = 2dx$

$$= \frac{\pi^x}{\ln(\pi)} + \frac{3}{2}\arcsin(u) + C = \boxed{\frac{\pi^x}{\ln(\pi)} + \frac{3}{2}\arcsin(2x) + C}$$

(c) $\frac{1}{3}\int 3x^8\sqrt[3]{x^3-1}\,dx$

$u = x^3 - 1$

$du = 3x^2\,dx$

$x^6 = (u+1)^2$

$$= \frac{1}{3}\int (u+1)^2\sqrt[3]{u}\,du = \frac{1}{3}\int (u^2+2u+1)u^{1/3}\,du$$

$$= \frac{1}{3}\int\left(u^{7/3} + 2u^{4/3} + u^{1/3}\right)du$$

$$= \frac{1}{3}\left(\frac{3u^{10/3}}{10} + \frac{6u^{7/3}}{7} + \frac{3u^{4/3}}{4}\right) + C$$

$$\boxed{= \frac{(x^3-1)^{10/3}}{10} + \frac{2(x^3-1)^{7/3}}{7} + \frac{(x^3-1)^{4/3}}{4} + C}$$

2. **[11 points]** Fungo is running along the number line.

At time $t = 0$, his **velocity** is $-\sqrt{3}$ ft/s.

After t seconds, his **acceleration** is $a(t) = \sec^2\left(\frac{t}{3}\right)$ ft/s^2.

From time $t = 0$ to $t = 4$, what is the **total distance** traveled by Fungo?

(Leave your answer in exact form. Please don't convert to decimal.)

$$v(t) = \int \sec^2\left(\frac{t}{3}\right) dt = \int 3\sec^2(u)\,du = 3\tan(u) + C$$

$u = \frac{t}{3}$

$du = \frac{1}{3} dt$

$$v(t) = 3\tan\left(\frac{t}{3}\right) + C$$

$$v(0) = 3\tan(0) + C = -\sqrt{3}$$

$$C = -\sqrt{3}$$

$$v(t) = 3\tan\left(\frac{t}{3}\right) - \sqrt{3}$$

When does he turn around? $3\tan\left(\frac{t}{3}\right) - \sqrt{3} = 0 \rightarrow \tan\left(\frac{t}{3}\right) = \frac{\sqrt{3}}{3}$

$\rightarrow \frac{t}{3} = \frac{\pi}{6} \longrightarrow t = \frac{\pi}{2}$

Position: $s(t) = \int \left(3\tan\left(\frac{t}{3}\right) - \sqrt{3}\right) dt = \int 9\tan(u)\,du - \sqrt{3}t + C$

$u = \frac{t}{3}$

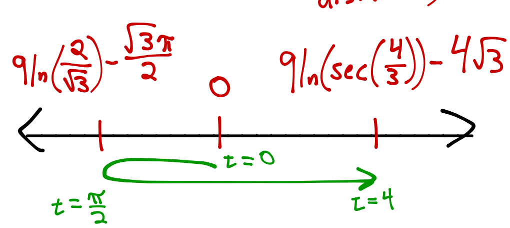
$du = \frac{1}{3} dt$

$= 9\ln\left|\sec\left(\frac{t}{3}\right)\right| - \sqrt{3}t + C$ 0 say (it doesn't matter, we just want distance)

$s(0) = 9\ln|\sec(0)| - \sqrt{3}(0) = 0$

$s\left(\frac{\pi}{2}\right) = 9\ln\left|\sec\left(\frac{\pi}{6}\right)\right| - \frac{\sqrt{3}\pi}{2} = 9\ln\left(\frac{2}{\sqrt{3}}\right) - \frac{\sqrt{3}\pi}{2}$

$s(4) = 9\ln\left(\sec\left(\frac{4}{3}\right)\right) - 4\sqrt{3}$



So, distance $= \boxed{\left(\frac{\sqrt{3}\pi}{2} - 9\ln\left(\frac{2}{\sqrt{3}}\right)\right) + \left(9\ln\left(\sec\left(\frac{4}{3}\right)\right) - 4\sqrt{3} - \left(9\ln\left(\frac{2}{\sqrt{3}}\right) - \frac{\sqrt{3}\pi}{2}\right)\right)}$

3. **[6 points]** Compute the following integral.

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{3}{n} \sin\left(2 + \frac{3i}{n}\right)$$

$\Delta x = \frac{b-a}{n} = \frac{3}{n}$

$x_i = a + i\Delta x = 2 + \frac{3i}{n}$

$a=2,\ b=5,\ f(x)=\sin(x)$

$= \int_2^5 \sin(x)\,dx$

$= -\cos(x)\Big]_2^5 = \boxed{\cos(2) - \cos(5)}$

4. **[6 points]** Let $h(x) = \int_{\ln(x)}^{3x} \cos(t^2+1)\,dt$. Find $h'(5)$.

Let $g(x) = \int_0^x \cos(t^2+1)\,dt$. So $g'(x) = \cos(x^2+1)$

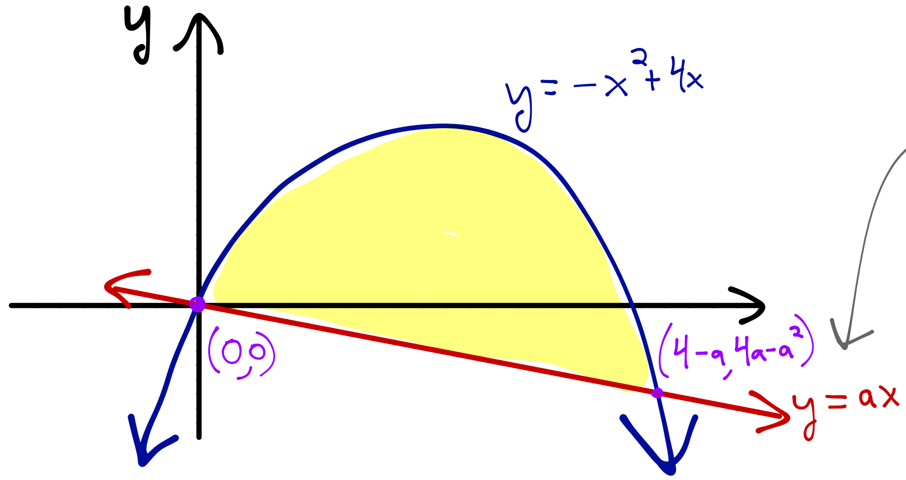
Now, $h(x) = g(3x) - g(\ln(x))$,

so $h'(x) = 3g'(3x) - \frac{1}{x}g'(\ln(x))$

$h'(x) = 3\cos(9x^2+1) - \frac{1}{x}\cos((\ln(x))^2+1)$

So $h'(5) = \boxed{3\cos(226) - \frac{1}{5}\cos((\ln(5))^2+1)}$

5. **[10 points]** Let R be the region bounded by the curves $y = -x^2 + 4x$ and $y = ax$. Find a constant $a < 4$ so that the area of R is 15.



(turns out the slope is negative, but ok to draw it the other way.)

Intersection? $-x^2 + 4x = ax$

$$-x^2 + (4-a)x = 0$$

$$x(4-a-x) = 0$$

$x = 0$ $\quad$ $x = 4-a$

Area is $\displaystyle\int_0^{4-a} \left(-x^2 + 4x - ax\right) dx$

$$= \int_0^{4-a} \left(-x^2 + (4-a)x\right) dx = \left(\frac{-x^3}{3} + \frac{(4-a)x^2}{2}\right)\Bigg]_0^{4-a}$$

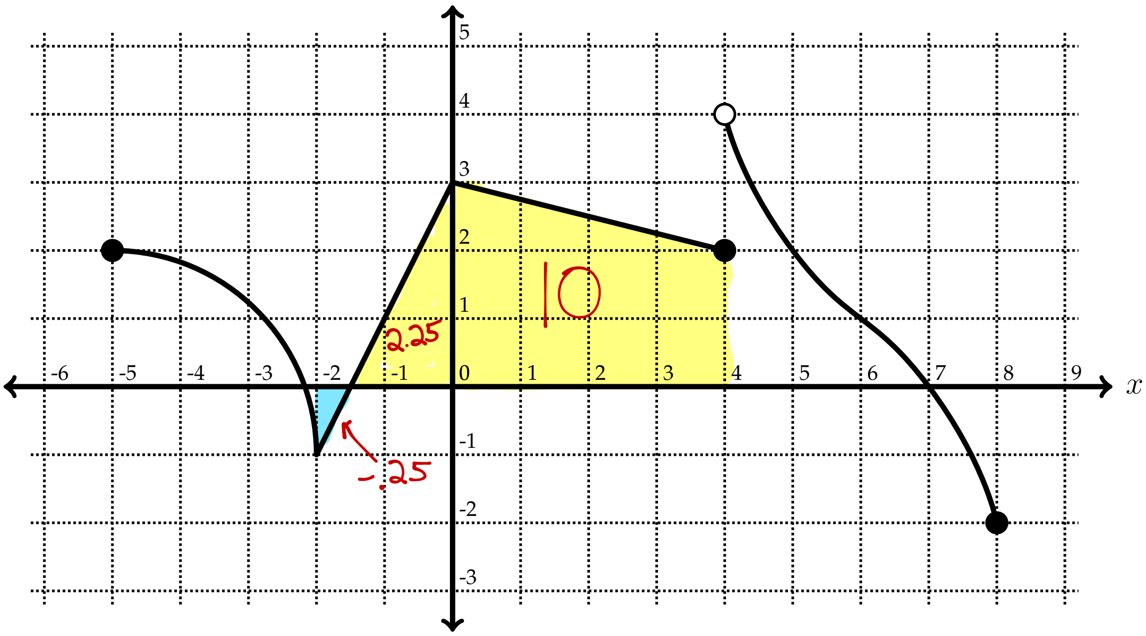
$$= \frac{-(4-a)^3}{3} + \frac{(4-a)^3}{2} = \frac{(4-a)^3}{6} = 15$$

$$(4-a)^3 = 90$$

$$4-a = \sqrt[3]{90}$$

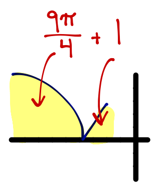
$$a = \boxed{4 - \sqrt[3]{90}}$$

6. **[4 points per part]** Behold: a graph!



Use this graph to answer the following questions.

(a) Compute $\displaystyle\int_{-5}^{-1} f(x)\,dx$.



If the graph were one unit higher, it would be easy:

But this is one unit lower, losing an area of 4.

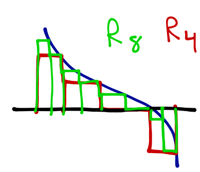
So $\displaystyle\int_{-5}^{-1} f(x)dx = \frac{9\pi}{4} + 1 - 4 = \boxed{\frac{9\pi}{4} - 3}$

(b) Compute $\displaystyle\int_{-1}^{1} f(1-3x)\,dx$. $= \displaystyle\frac{-1}{3}\int_{4}^{-2} f(u)du = \frac{1}{3}\int_{-2}^{4} f(u)du = \frac{1}{3}(12) = \boxed{4}$

$u = 1-3x$
$du = -3\,dx$

(c) Consider the integral $\displaystyle\int_{4}^{8} f(x)\,dx$. Which approximation is larger: R_4 or R_8? Explain.

R_4 & R_8 are both underestimates since f is decreasing, but R_8 is closer to the actual value. So $\boxed{R_8 \text{ is greater.}}$

(Or, draw rectangles: 

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