Math 126 E - Spring 2016
Midterm Exam Number Two
May 17, 2016

Name: ______________________ Student ID no. : ______________

Signature: ____________________________ Section: __________

1	12	
2	12	
3	8	
4	13	
5	8	
6	7	
Total	60	

- This exam consists of SIX problems on SIX pages, including this cover sheet.
- Show all work for full credit. Show no work for zero credit.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- Write all of your work on the exam itself. If you use the back of the page, please indicate that you have done so!
- You may use a TI-30X IIS on this exam. No other electronic devices are allowed.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 50 minutes to complete the exam.

–

1. **[12 points]** A particle has an initial velocity of $\langle 0,0,0\rangle$, and after t seconds its acceleration is given by $\mathbf{a}(t) = \langle 6t^2, -3, 9\sqrt{t}\rangle$.

Compute the **tangential** and **normal** components of acceleration after one second.

(Please specify which is which!)

First, antidifferentiate:

$$\vec{v}(t) = \langle 2t^3 + C_1, -3t + C_2, 6t^{3/2} + C_3 \rangle$$

$C_1 = C_2 = C_3 = 0$, since $\vec{v}(0) = \langle 0,0,0\rangle$

$$\vec{v}(1) = \langle 2, -3, 6\rangle$$

$$\vec{a}(1) = \langle 6, -3, 9\rangle$$

$$a_T = \frac{\vec{v}(1)\cdot\vec{a}(1)}{|\vec{v}(1)|} = \frac{\langle 2,-3,6\rangle\cdot\langle 6,-3,9\rangle}{7} = \boxed{\frac{75}{7}}$$

$$a_N = \frac{|\vec{v}(1)\times\vec{a}(1)|}{|\vec{v}(1)|} = \frac{|\langle -9, 18, 12\rangle|}{7} = \boxed{\frac{3\sqrt{61}}{7}}$$

2. Consider the surface $4x^2 - 2xy + y^2z - y^3 = 44$.

(a) **[10 points]** Find the equation for the tangent plane to this surface at the point $(3, 4, 6)$.

$\frac{\partial}{\partial x}$ (treating y as constant)

$$8x - 2y + y^2 \frac{\partial z}{\partial x} = 0 \rightarrow \frac{\partial z}{\partial x} = \frac{2y - 8x}{y^2} \xrightarrow{x=3,\ y=4} \frac{\partial z}{\partial x} = -1$$

$\frac{\partial}{\partial y}$ (treating x as constant):

$$-2x + 2yz + y^2 \frac{\partial z}{\partial y} - 3y^2 = 0 \rightarrow \frac{\partial z}{\partial y} = \frac{3y^2 + 2x - 2yz}{y^2}$$

$x=3$ $y=4$ $z=6$

$$\frac{\partial z}{\partial y} = \frac{5}{16}$$

Equation for tangent plane

$$\boxed{z - 6 = -1(x-3) + \frac{5}{16}(y-4)}$$

(b) **[2 points]** Use linearization to approximate a constant b such that the point $(2.98, b, 5.96)$ lies on the surface.

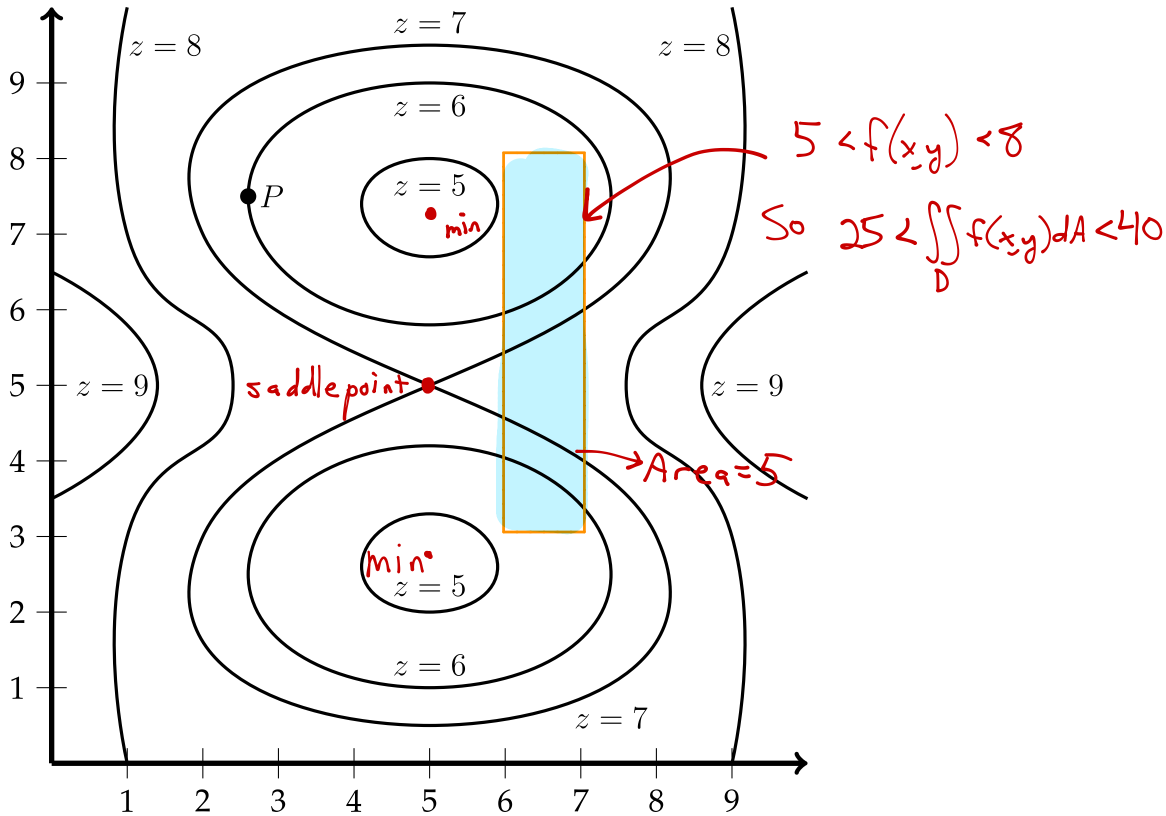
$x = 2.98$
$z = 5.96$

$$5.96 - 6 = -1(2.98 - 3) + \frac{5}{16}(y-4)$$

$$-0.06 = \frac{5}{16}(y-4)$$

$$\boxed{y = 3.808}$$

3. $f(x, y)$ is a smooth continuous function whose level curves are shown below.



Use this graph to answer the following questions. *You do not need to show work.*

(a) **[1 point each]** At the point P, indicate whether the following partial derivatives are positive, negative, or zero. (Circle your answers.)

$\frac{\partial z}{\partial x}$: Positive Negative Zero

$\frac{\partial^2 z}{\partial x^2}$: Positive Negative Zero

$\frac{\partial z}{\partial y}$: Positive Negative Zero

$\frac{\partial^2 z}{\partial y^2}$: Positive Negative Zero

(b) **[2 points]** $f(x, y)$ has three critical points. Estimate their coordinates, and classify them as local maxima, local minima, or saddlepoints.

(5, 7.5) & (5, 2.5): local minima (5, 5): saddlepoint

(c) **[2 points]** Consider the double integral $\int_3^8 \int_6^7 f(x, y)\, dx\, dy$. Which of the following correctly estimates that integral?

(Circle one.)

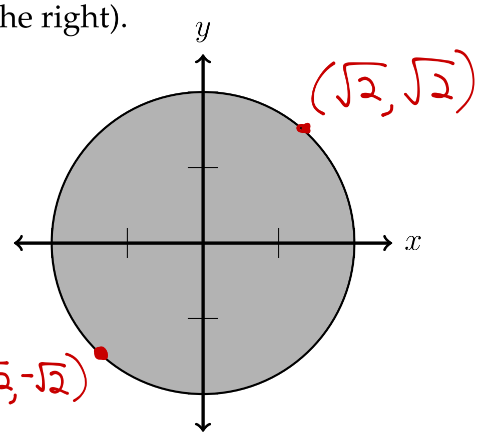
Between 0 and 20. Between 20 and 40. Between 40 and 60.

Between 60 and 80. Between 80 and 100. Greater than 100.

4. **[13 points]** Compute the absolute maximum and minimum values of the function

$$f(x, y) = e^{x+y}(x^2 + y^2)$$

on the disk of radius 2 centered at the origin (pictured to the right).



First, where do the partial derivatives $= 0$?

$$f_x(x,y) = e^{x+y}(x^2+y^2) + e^{x+y}(2x)$$
$$= e^{x+y}(x^2+2x+y^2) = 0$$

$$f_y(x,y) = e^{x+y}(x^2+y^2) + e^{x+y}(2y)$$
$$= e^{x+y}(x^2+y^2+2y) = 0$$

If $x^2+y^2+2x = x^2+y^2+2y$, then $x = y$.

So $x^2+x^2+2x = 0 \rightarrow 2x(x+1) = 0$

$x=0,\ y=0$

or

$x=-1,\ y=-1$

Points to check:

$(0,0)$

$(-1,-1)$

$(-\sqrt{2},-\sqrt{2})$

$(\sqrt{2},\sqrt{2})$

On the boundary, $x^2+y^2=4$, so $f(x,y) = 4e^{x+y}$

big if $x+y$ is big (top-right corner),

small if $x+y$ is small (lower-left)

Check!

$f(0,0) = \boxed{0}$

$f(-1,-1) = \frac{1}{e^2}$

abs min

$f(\sqrt{2},\sqrt{2}) = 4e^{2\sqrt{2}} \approx \boxed{67.68}$

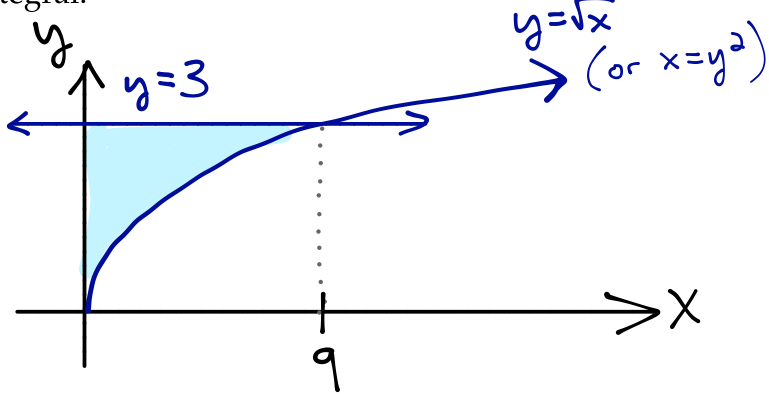
$f(-\sqrt{2},-\sqrt{2}) = 4e^{-2\sqrt{2}} \approx 0.24$

abs max

5. **[8 points]** Compute the following integral.

$$\int_0^9 \int_{\sqrt{x}}^3 \cos(\pi y^3)\, dy\, dx$$

Yikes! Let's draw it:



Change order?

y goes from 0 to 3.

x goes from 0 to y^2.

$$\int_0^3 \int_0^{y^2} \cos(\pi y^3)\, dx\, dy = \int_0^3 \left(x\cos(\pi y^3) \Big]_0^{y^2} \right) dy = \frac{1}{3\pi}\int_0^3 3\pi y^2 \cos(\pi y^3)\, dy$$

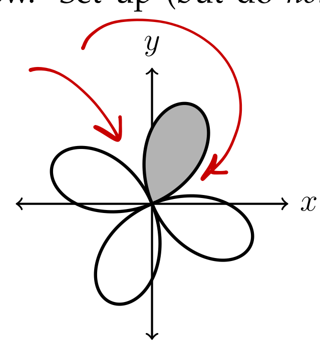
$u = \pi y^3$

$du = 3\pi y^2 dy$

$$= \frac{1}{3\pi}\int_0^{27\pi} \cos(u)\, du = \frac{1}{3\pi}\left(\sin(u)\right)\Big]_0^{27\pi} = \boxed{0}$$

6. **[7 points]** Consider the rose $r = \sin(2\theta) - \cos(2\theta)$, shown below. Set up (but do *not* evaluate) an integral to find the area of one petal of the rose.

What θ's are these?



$\sin(2\theta) - \cos(2\theta) = 0$

$\sin(2\theta) = \cos(2\theta)$

$\tan(2\theta) = 1$

$2\theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

$\theta = \frac{\pi}{8}$ or $\frac{5\pi}{8}$

$$\boxed{\int_{\pi/8}^{5\pi/8} \int_0^{\sin(2\theta)-\cos(2\theta)} r\, dr\, d\theta}$$