Math 125 F - Winter 2016
Midterm Exam Number One
January 28, 2016

Name: ____________________ Student ID no. : ____________

Signature: ________________________ Section: __________

1	12	
2	12	
3	10	
4	5	
5	12	
6	9	
Total	60	

- This exam consists of SIX problems on SIX pages, including this cover sheet.
- Show all work for full credit.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you run out of room, write on the back of the page, but *indicate that you have done so*!
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You may use a *scientific calculator*. Calculators with graphing, differentiation, integration, or algebraic capabilities are not allowed.
- You have 80 minutes to complete the exam.

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1. **[4 points per part]** Compute the indefinite integrals.

(a) $\int \left(\sqrt[7]{x} - \frac{2}{\sqrt{1-x^2}} \right) dx$

$x^{1/7}$

$$= \boxed{\frac{7x^{8/7}}{8} - 2\arcsin(x) + C}$$

(b) $\int (x^{1.7} + e^{3x})\, dx$

$$\frac{x^{2.7}}{2.7} + \frac{1}{3}\int 3e^{3x}\,dx = \frac{x^{2.7}}{2.7} + \frac{1}{3}\int e^{u}\,du = \boxed{\frac{x^{2.7}}{2.7} + \frac{e^{3x}}{3} + C}$$

$u = 3x$

$du = 3\,dx$

(c) $\int \frac{\sin^2(\ln(x))\cos(\ln(x))}{x}\, dx = \int u^2\,du$

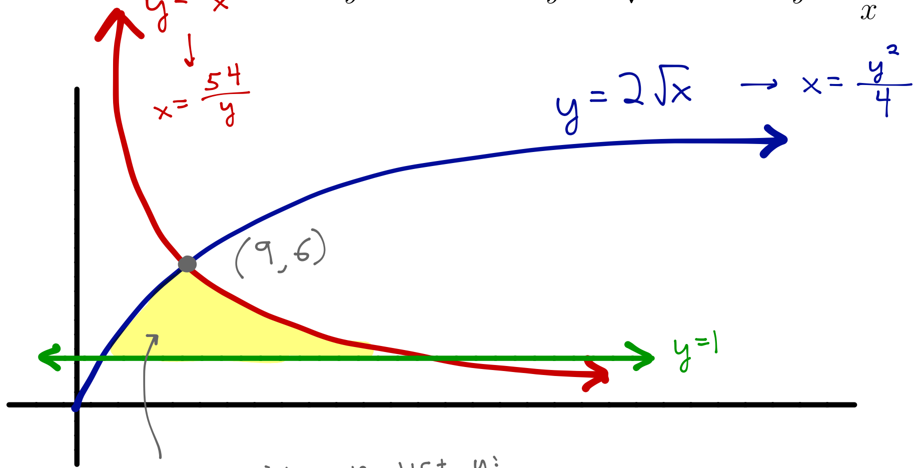
$u = \sin(\ln(x))$

$du = \cos(\ln(x)) \cdot \frac{1}{x}\,dx$

$$= \frac{u^3}{3} + C = \boxed{\frac{\sin^3(\ln(x))}{3} + C}$$

2. **[12 points]** Compute the area of the region bounded by the following three curves:

$$y = 1 \qquad y = 2\sqrt{x} \qquad y = \frac{54}{x}$$



easier to integrate w.r.t. y:

$$\int_1^6 \left(\frac{54}{y} - \frac{y^2}{4}\right)\Bigg]_1^6$$

$$= \left(54\ln|y| - \frac{y^3}{12}\right)\Bigg]_1^6$$

$$= \left(54\ln(6) - \frac{216}{12}\right) + \left(\frac{1}{12}\right)$$

$$= \boxed{54\ln(6) - \frac{215}{12}}$$

3. **[10 points]** A remote-controlled tomato is moving along the number line. Its velocity after t seconds is given by the formula

$$v(t) = 9 - 3^t.$$

Compute the total distance traveled by the tomato from time $t = 0$ to $t = 4$.

(You do not need to simplify your answer.)

When does it turn around? $v(t) = 9 - 3^t = 0 \downarrow t = 2$

Displacement after t seconds:

$$s(t) = \int_0^t 9 - 3^x \, dx$$

$$= \left(9x - \frac{3^x}{\ln(3)}\right)\Big]_0^t$$

$$= 9t - \frac{3^t - 1}{\ln(3)}$$

when it turns around

$$s(0) = 0$$

$$s(2) = 18 - \frac{8}{\ln(3)}$$

$$s(4) = 36 - \frac{80}{\ln(3)}$$

$$\text{Total distance} = \left(18 - \frac{8}{\ln(3)} - 0\right) + \left(\left(18 - \frac{8}{\ln(3)}\right) - \left(36 - \frac{80}{\ln(3)}\right)\right)$$

$$= \boxed{\frac{64}{\ln(3)}}$$

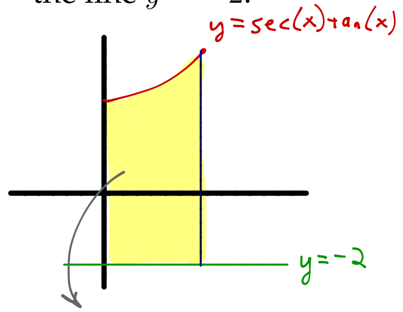
4. **[5 points]** Write (but do not simplify) a formula for the L_{1000} approximation of $\int_0^2 \sin(x)\,dx$.

(*Please* use Σ-notation. Do not write out a thousand summands.)

$$\sum_{i=0}^{999} f(a + i\Delta x)\Delta x = \sum_{i=0}^{999} \sin\left(\frac{2i}{1000}\right) \cdot \frac{2}{1000} = \boxed{\sum_{i=0}^{999} \sin\left(\frac{i}{500}\right)\left(\frac{1}{500}\right)}$$

5. **[12 points]** Let $\mathcal{R}$ be the region in the x-y plane below $y = \sec(x)\tan(x)$ and above $y = -2$ from $x = 0$ to $x = \frac{\pi}{4}$.

(a) Write an integral to compute the volume of the solid formed by revolving $\mathcal{R}$ around the line $y = -2$.



$$V = \int_0^{\frac{\pi}{4}} \pi\left(\sec(x)\tan(x) + 2\right)^2 dx$$

(b) Evaluate the integral from part (a).

$$= \pi\int\left(\sec^2(x)\tan^2(x) + 4\sec(x)\tan(x) + 4\right)dx$$

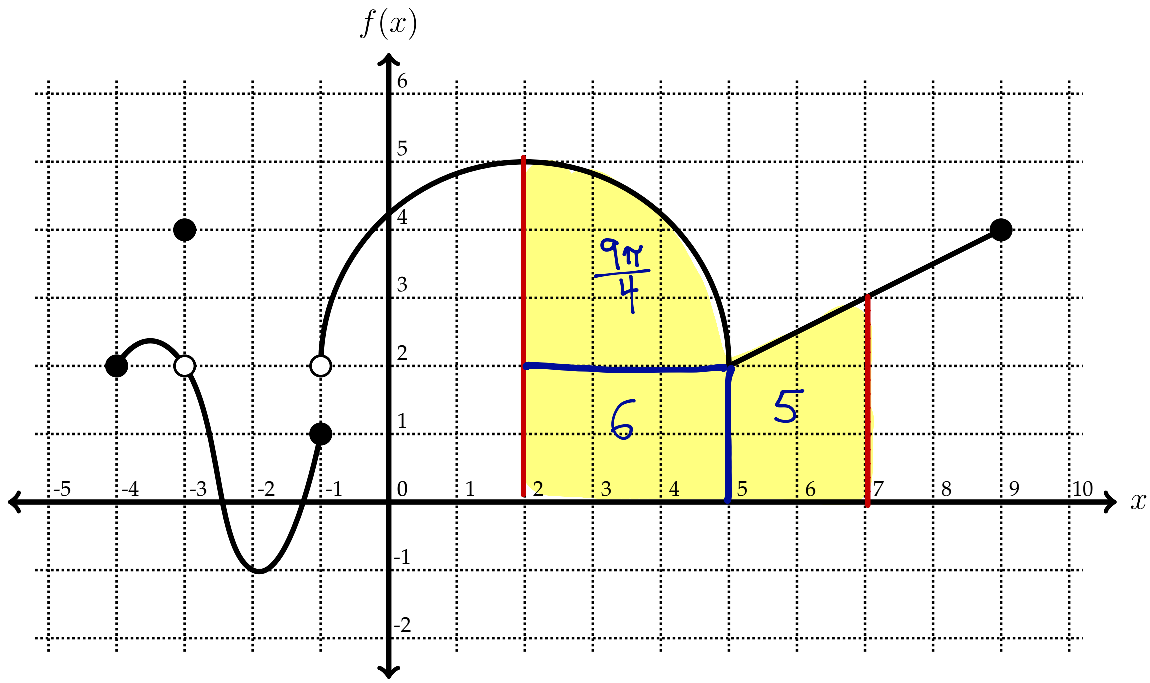
$u = \tan(x)$
$du = \sec^2(x)$,
so this is $u^2 du$,
integral is $\frac{\tan^3(x)}{3}$

$$= \pi\left(\frac{\tan^3(x)}{3} + 4\sec(x) + 4x\right)\Big]_0^{\frac{\pi}{4}}$$

$$= \pi\left(\left(\frac{1^3}{3} + 4\sqrt{2} + \pi\right) - \left(0 + 4 + 0\right)\right)$$

$$= \boxed{\pi\left(\frac{-11}{3} + 4\sqrt{2} + \pi\right)}$$

6. Below is the graph of $f(x)$, the most beautiful function you've ever seen.



Use this graph to answer the following questions.

(a) **[3 points]** Does $\int_{-4}^{-1} f(x)\,dx$ exist? Explain, briefly.

$\boxed{\text{Yeah!}}$ It only has one (removable) discontinuity, so the Riemann sums converge.

(b) **[3 points]** Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} f\left(2 + \frac{5i}{n}\right)\frac{5}{n}$.

$$\int_2^7 f(x)dx = \boxed{11 + \frac{9\pi}{4}}$$

(c) **[3 points]** Let $h(x) = \int_0^{2x} f(3t)\,dt$. Compute $h'(1)$.

Let $g(x) = \int_0^x f(3t)dt$. Then $g'(x) = f(3x)$.

$h(x) = g(2x)$, so $h'(x) = g'(2x)\cdot 2 = 2f(6x)$

$$h'(1) = 2f(6) = \boxed{5}$$