# Math 120 A - Autumn 2016
# Midterm Exam Number Two
# November 17th, 2016

Name: ______________________________ Student ID no. : ____________________

Signature: ____________________________________ Section: ______________

| | | |
|---|---|---|
| 1 | 14 | |
| 2 | 15 | |
| 3 | 15 | |
| 4 | 16 | |
| Total | 60 | |

- This exam consists of FOUR problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic device are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- Draw a box around your final answer to each problem.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 50 minutes to complete the exam.

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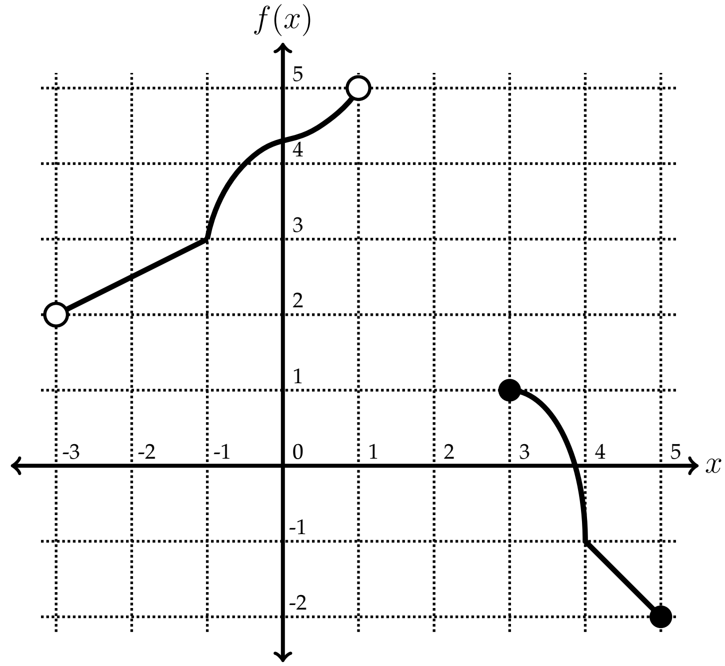
1. Happy Thursday! I bought you this graph.




(a) **[4 points]** Compute $f(f(f(4)))$.

$$f(f(f(4))) = f(f(-1)) = f(3) = \boxed{1}$$

(b) **[5 points]** Find the domain and range of $f^{-1}(x)$.

Domain of $f^{-1}$ = range of $f$ = $\boxed{[-2,1] \cup (2,5)}$

Range of $f^{-1}$ = domain of $f$ = $\boxed{(-3,1) \cup [3,5]}$

(c) **[5 points]** Let $g(x) = f(2x+1)+1$. Find the domain and range of $g(x)$.

| | Domain | Range |
|---|---|---|
| Original | $(-3,1)\cup[3,5]$ | $[-2,1]\cup(2,5)$ |
| Shift up 1 | $(-3,1)\cup[3,5]$ | $[-1,2]\cup(3,6)$ |
| Shift left 1 | $(-4,0)\cup[2,4]$ | $[-1,2]\cup(3,6)$ |
| Scale horiz. by ½ | $\boxed{(-2,0)\cup[1,2]}$ | $\boxed{[-1,2]\cup(3,6)}$ |

2. **[15 points]** Gomba is on a diet. His weight is a linear-to-linear rational function of time.

Right now, Gomba weighs 21 pounds.

In 1 month, he will weigh 20.5 pounds.

In 7 months, he will weigh 18.75 pounds.

In the long run, what will Gomba's weight approach?

(Assume Gomba will live forever.)

$$f(x) = \frac{ax+b}{x+d}$$

Gomba's weight after x months

Want h. asymptote:

$$\boxed{15.6 \text{ lbs}}$$

$$f(0) = \frac{b}{d} = 21 \longrightarrow b = 21d$$

$$f(1) = \frac{a+b}{1+d} = 20.5 \longrightarrow a+b = 20.5 + 20.5d$$

$$f(7) = \frac{7a+b}{7+d} = 18.75 \longrightarrow 7a+b = 131.25 + 18.75d$$

$$a = 20.5 + 20.5d - 21d$$

$$7(20.5 - 0.5d) + 21d = 131.25 + 18.75d$$

$$12.25 = 1.25d$$

$$d = 9.8$$

$$b = 21(9.8)$$

$$b = 205.8$$

$$a = 20.5 - .5(9.8)$$

$$a = 15.6$$

3. **[5 points per part]** The rent for a one-bedroom apartment in Beattle is growing exponentially. (Even though the city is filled with bees.)

   (a) In the year 2000, the rent in Beattle was $\$1020$, and it increases by $2.3\%$ per year.

   Write a function $f(t)$ for the rent in Beattle $t$ years after 2000.

$$f(t) = A_0 b^t \qquad b = 1 + .023$$

$$\boxed{f(t) = 1020(1.023)^t}$$

   (b) The average monthly rent in Tickoma is also growing exponentially.

   In the year 2007, the rent in Tickoma was $\$500$ less than the rent in Beattle.

   In the year 2016, the rent in Tickoma is $\$1000$.

   Write a function $g(t)$ for the rent in Tickoma $t$ years after 2000.

$$g(t) = A_0 b^t$$

$$g(7) = A_0 b^7 = f(7) - 500 = 696$$

$$g(16) = A_0 b^{16} = 1000 \quad \text{divide}$$

$$b^9 = \frac{1000}{696}$$

$$b = \left(\frac{1000}{696}\right)^{1/9} = 1.04109$$

$$A_0 (1.04109)^7 = 696 \rightarrow A_0 = 525$$

$$\boxed{g(t) = 525(1.04109)^t}$$

   (c) When will the rents in Beattle and Tickoma be equal?

   (Round your answer to the nearest year.)

$$1020(1.023)^t = 525(1.04109)^t$$

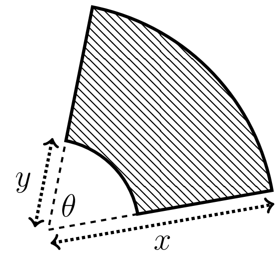
$$\ln(1020) + \ln(1.023^t) = \ln(525) + \ln(1.04109^t)$$

$$\ln(1020) + t\ln(1.023) = \ln(525) + t\ln(1.04109)$$

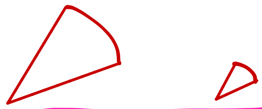
$$t(\ln(1.023) - \ln(1.04109)) = \ln(525) - \ln(1020)$$

$$t = \frac{\ln(525) - \ln(1020)}{\ln(1.023) - \ln(1.04109)} \approx \boxed{38 \text{ years}}$$

4. A *polar rectangle* is the region bounded by two concentric circular arcs and two rays through the center of those arcs. Okay, fine, here's a picture:




(a) **[4 points]** Write a formula for the *area* of this polar rectangle.
(Your answer will involve $x$, $y$, and $\theta$. Let $\theta$ be measured in radians.)

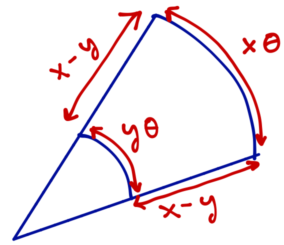


$$A = \boxed{\frac{1}{2}\theta x^2 - \frac{1}{2}\theta y^2}$$

(b) **[4 points]** Write a formula for the *perimeter* of this polar rectangle.

$$P = (x-y) + (x-y) + x\theta + y\theta$$

$$P = \boxed{(2+\theta)x + (-2+\theta)y}$$




(c) **[8 points]** Suppose you have 24 meters of fencing, and you want to construct a fence in the shape of a polar rectangle with central angle $\theta = 1.2$ radians. What is the maximum possible area of your fence?

$$24 = (2+1.2)x + (-2+1.2)y$$

$$24 = 3.2x - 0.8y$$

$$y = 4x - 30$$

$$A = \frac{1}{2}(1.2)x^2 - \frac{1}{2}(1.2)(4x-30)^2$$

$$A = 0.6x^2 - 0.6(16x^2 - 240x + 900)$$

$$A = -9x^2 + 144x - 540$$

Graph!

$$k = -540 - \frac{144^2}{4(-9)} = \boxed{36 \text{ sq. m}}$$