

Math 120 A - Autumn 2016
Midterm Exam Number One
October 20th, 2016

Name: _____

Student ID no. : _____

Signature: _____

Section: _____

1	15	
2	15	
3	8	
4	8	
5	14	
Total	60	

- This exam consists of FIVE problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic device are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

1. [15 points] Candela stands 10 meters west and 16 meters south of a gym. Spark stands 3 meters east and 11 meters north of the gym.

Candela walks due east until she is 17.8 meters away from the gym. Then, she turns and walks in a straight line towards Spark.

How close does Candela get to the gym?

Where is this?
 Intersection of $y = -16$, $x^2 + y^2 = 17.8^2$
 $x^2 + (-16)^2 = 17.8^2$
 $x = \pm 7.8$
 left side of circle

Need perpendicular to path.
 $y = \frac{5}{2}(x-3)+11$ $y = \frac{-2}{5}x$

$$\frac{5}{2}(x-3)+11 = \frac{-2}{5}x$$

$$25x - 75 + 110 = -4x$$

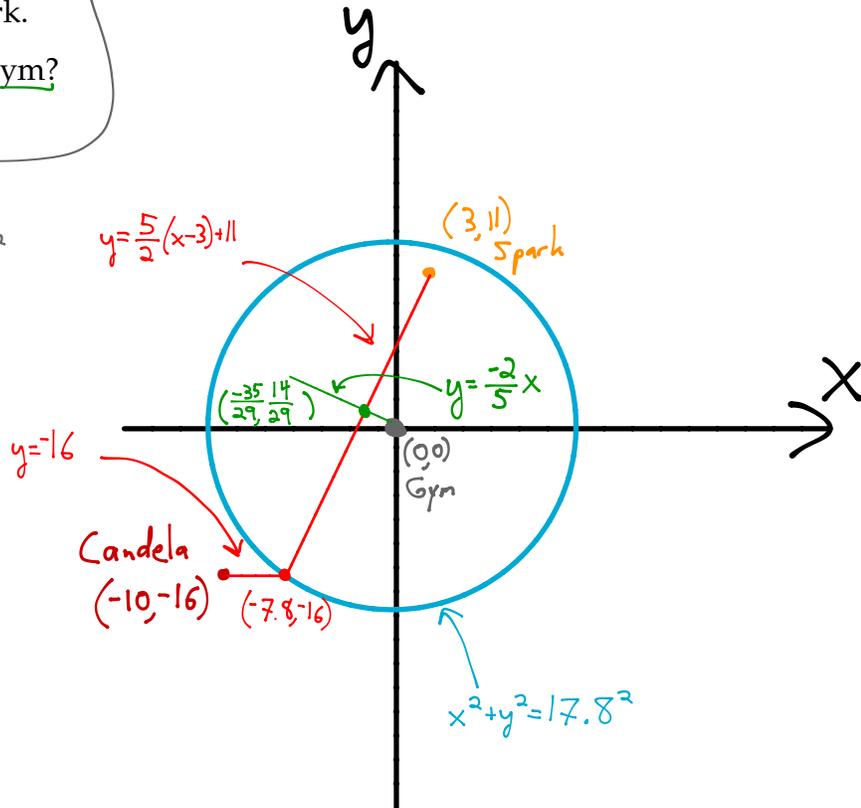
$$29x = -35$$

$$x = \frac{-35}{29}$$

$$y = \frac{-2}{5} \left(\frac{-35}{29} \right) = \frac{14}{29}$$

So closest point is $\left(\frac{-35}{29}, \frac{14}{29} \right)$ and distance to origin is:

$$d = \sqrt{\left(\frac{-35}{29} \right)^2 + \left(\frac{14}{29} \right)^2} \approx 1.2999 \text{ meters}$$



2. [5 points per part] Luke and Reva begin walking in the xy -plane at constant speeds at the same time.

Luke walks from $(3, 5)$ to $(-2, 4)$ in a straight line, reaching it in 10 seconds.

Reva walks from $(-4, 6)$ in a straight line. When Luke crosses the y -axis, Reva is at $(4, 1)$.

- (a) Write parametric equations for Luke's position, t seconds after he starts walking.

$$\begin{aligned} x_0 &= 3 & y_0 &= 5 \\ x_1 &= -2 & y_1 &= 4 \\ \Delta x &= -5 & \Delta y &= -1 \\ \Delta t &= 10 \end{aligned}$$

$$\begin{cases} x = 3 - \frac{5}{10}t \\ y = 5 - \frac{1}{10}t \end{cases}$$

- (b) Write parametric equations for Reva's position, t seconds after she starts walking.

When is this? $x = 3 - \frac{5}{10}t = 0$
 $3 = \frac{1}{2}t$
 $t = 6$

$$\begin{aligned} x_0 &= -4 & y_0 &= 6 \\ x_1 &= 4 & y_1 &= 1 \\ \Delta x &= 8 & \Delta y &= -5 \\ \Delta t &= 6 \end{aligned}$$

$$\begin{cases} x = -4 + \frac{8}{6}t \\ y = 6 - \frac{5}{6}t \end{cases}$$

- (c) When is Luke directly east of Reva?

y -coordinates are equal:

$$5 - \frac{1}{10}t = 6 - \frac{5}{6}t$$

$$\left(\frac{5}{6} - \frac{1}{10}\right)t = 1$$

$$\frac{44}{60}t = 1$$

$$t = \frac{60}{44} = \frac{15}{11} \text{ seconds}$$

3. [8 points] Consider the following multipart function f :

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x + 5 & \text{if } 0 < x < 4 \\ x^2 + 6 & \text{if } x \geq 4 \end{cases}$$

Find all solutions to the equation $f(x) = x^2 - 1$.

Handwritten work for solving $f(x) = x^2 - 1$:

If $x \leq 0$:

$$0 = x^2 - 1$$

$$1 = x^2$$

~~$x = 1$~~
or
 $x = -1$

If $0 < x < 4$:

$$x + 5 = x^2 - 1$$

$$0 = x^2 - x - 6$$

$$0 = (x - 3)(x + 2)$$

$x = 3$
or
 ~~$x = -2$~~

If $x \geq 4$:

$$x^2 + 6 = x^2 - 1$$

$$6 = -1$$

Nope!

Two solutions: $x = -1$ or 3

4. [8 points] Find all values of d such that the vertex of $y = \underbrace{d}_{a}x^2 + \underbrace{5}_{b}x + \underbrace{d+1}_{c}$ is on the x -axis. $k=0$:

$$c - \frac{b^2}{4a} = 0$$

$$d + 1 - \frac{5^2}{4d} = 0$$

$$4d(d + 1) = \left(\frac{25}{4d}\right)4d$$

$$4d^2 + 4d = 25$$

$$4d^2 + 4d - 25 = 0$$

$$d = \frac{-4 \pm \sqrt{16 - 4(4)(-25)}}{8} \approx \begin{cases} -3.0495 \\ \text{or} \\ 2.0495 \end{cases}$$

5. [14 points] Ken sells sweaters. His profit is a quadratic function of the price he charges.

If he gives the sweaters away for free, he will **lose** \$100.

If he charges \$10 per sweater, he will **earn** \$80.

If he charges \$20 per sweater, he will **earn** \$180.

How much will he earn by charging \$33 per sweater?

profit $\rightarrow y = ax^2 + bx + c$
price

$$\begin{aligned} x=0, y=-100 &: \quad -100 = c \\ x=10, y=80 &: \quad 80 = 100a + 10b + c \\ x=20, y=180 &: \quad 180 = 400a + 20b + c \end{aligned}$$
$$\begin{aligned} 80 &= 100a + 10b - 100 \rightarrow 10b = 180 - 100a \rightarrow b = 18 - 10a \\ 180 &= 400a + 20b - 100 \\ 180 &= 400a + 20(18 - 10a) - 100 \\ -80 &= 200a \\ a &= -0.4 \\ b &= 18 - 10(-0.4) \\ b &= 22 \end{aligned}$$

So: $y = -0.4x^2 + 22x - 100$

And when $x=33$, $y = -0.4(33)^2 + 22(33) - 100 = 190.4$

So: $\$190.40$