Math 126: Calc III **Spring 2015**

A List of Topics for the Final

Here's a list of things you should be comfortable doing for the final exam.

Older Stuff

1. **Three-Dimensional Coordinate Systems (Chapter 12.1)**
   (a) Plot points in three dimensions.
   (b) Compute the distance between two points in $\mathbf{R}^3$.
   (c) Recognize equations for cylinders and spheres.
2. **Vectors (Chapter 12.2)**
   (a) Recognize vectors written in a variety of forms.
   (b) Find a vector from one point to another.
   (c) Add, subtract, and scale vectors, either geometrically or algebraically.
   (d) Compute the length of a vector.
3. **The Dot Product (Chapter 12.3)**
   (a) Compute the dot product between two vectors.
   (b) Determine when two vectors are parallel or perpendicular.
   (c) Find the angle between two vectors.
   (d) Compute $\text{proj}_{\mathbf{a}}(\mathbf{b})$ and $\text{comp}_{\mathbf{a}}(\mathbf{b})$.
4. **The Cross Product (Chapter 12.4)**
   (a) Compute the cross product of two vectors in $\mathbf{R}^3$.
   (b) Understand the connection between the directions of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{a} \times \mathbf{b}$.
   (c) Find the area of a triangle or parallelogram using the cross product.
5. **Lines & Planes (Chapter 12.5)**
   (a) Find the equation for a line given a point and a direction vector.
   (b) Find the equation for a plane given a point and a normal vector.
   (c) Solve all sorts of problems involving lines & planes, including but not limited to:
      - Check whether two lines are parallel, intersecting, or skew.
      - Find the intersection of two planes.
      - Find the intersection of a line and a plane.
      - Find a plane through three points.
      - Find a plane through a point and a line.
      - Find the distance from a point to a plane.
      - Find the angle between two planes.

6. **Quadric Surfaces (Chapter 12.6)**
   (a) Complete the square to write the equation for a quadric surface in standard form.
   (b) Recognize various quadric surfaces from their equations.
   (c) Determine the shape of a quadric surface by drawing its traces.
   (d) Find the intersection(s) of a line with a quadric surface.
7. **Vector Functions and Space Curves (Chapter 13.1 & 10.1)**
   (a) Compute limits of vector functions.
   (b) Sketch the space curve of a vector function.
   (c) Check whether the space curves of two vector functions intersect, and if so where.
   (d) Locate the intersection of a space curve and a quadric surface.
   (e) Find a vector function to represent the intersection of two surfaces.
8. **Derivatives and Integrals of Vector Function (Chapter 13.2 & 10.2)**
   (a) Take the derivative of a vector function.
   (b) Find the tangent vector to a space curve at a given point.
   (c) Compute antiderivatives of vector functions.
   (d) Determine the arc length of a two-dimensional parametric curve.
9. **Polar Coordinates (Chapter 10.3)**
   (a) Convert points and equations between polar form and Cartesian form.
   (b) Find tangent lines to polar functions.
   (c) (Roughly) sketch polar functions.

Old Stuff

10. **Arc Length and Curvature (Chapter 13.3)**
    (a) Compute arc length for vector functions in three or more dimensions.
    (b) Find κ, $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$ for a given vector function.
    (c) Use $\mathbf{T}$ and $\mathbf{B}$ to find normal and osculating planes to a space curve.
11. **Velocity & Acceleration (Chapter 13.4)**
    (a) Compute velocity and acceleration vectors for an object using its position vector.
    (b) Integrate to find the position vector using the acceleration vector.
    (c) Apply the equation $\mathbf{F} = m\mathbf{a}$.
    (d) Decompose an acceleration vector into its normal and tangential components.
12. **Functions of Several Variables (Chapter 14.1)**
    (a) Find the domain of a function of two or more variables.
    (b) Analyze the level curves of a function of several variables.
    (c) Sketch a function of two variables, when possible.

13. **Partial Derivatives (Chapter 14.3)**

    (a) Compute the partial derivatives of a function of two or more variables.

    (b) Interpret those partial derivatives as slopes.

    (c) Find tangent vectors to a multivariable function at certain points.

    (d) Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

    (e) Compute higher derivatives, and apply Clairaut's theorem.

14. **Tangent Planes and Linear Approximations (Chapter 14.4)**

    (a) Find the tangent plane to a function of two variables at a given point.

    (b) Linearize a function at some point, and use it to approximate things.

    (c) Use differentials to estimate change in a multivariable function.

15. **Optimization (Chapter 14.7)**

    (a) Compute the critical points of a function of two variables.

    (b) Distinguish between local minima, local maxima, and saddlepoints.

    (c) Recognize local extrema conceptually, or using level curves.

    (d) Find the absolute maximum and minimum values of $f(x, y)$ over some domain.

16. **Double Integrals over Rectangles (Chapter 15.1)**

    (a) Estimate the volume under a surface using double Riemann sums.

    (b) Interpret double integrals as volumes, and compute familiar ones.

    (c) Find the average value of a function, based on this double integral.

17. **Iterated Integrals (Chapter 15.2)**

    (a) Use iterated integrals to compute the exact volume over a rectangular region.

    (b) Use Fubini's theorem to reverse the order of integration when necessary.

18. **Double Integrals over General Regions (Chapter 15.3)**

    (a) Find the volume under a surface over a non-rectangular region.

    (b) Set up iterated integrals based on a description of the region.

    (c) Reverse the order of integration, when necessary.

19. **Double Integrals in Polar Coordinates (Chapter 15.4)**

    (a) Recognize a double integral that would be more easily solved in polar coordinates.

    (b) Rewrite a double integral so that this computation is possible.

    (c) Set up a polar integral based on a description of the region.

New Stuff

20. **Mass and Center of Mass (Chapter 15.5)**

    (a) Compute the mass and center of mass of a lamina using appropriate double integrals.

21. **Linear Approximation (Taylor Notes §1)**

    (a) Compute the first Taylor polynomial of a function centered at some point.

    (b) Bound the error in your estimate by using the second derivative of the function.

22. **Quadratic Approximation (Taylor Notes §2)**

    (a) Compute the second Taylor polynomial of a function centered at some point.

    (b) Bound the error in your estimate by using the third derivative of the function.

23. **General Taylor Polynomials (Taylor Notes §3)**

    (a) Compute the nth Taylor polynomial of a function centered at some point.

    (b) Bound the error in your estimate by using the $(n+1)$st derivative of the function.

    (c) For a given f and b, find an interval in which the error is less than some value.

24. **Taylor Series (Taylor Notes §4)**

    (a) Find the Taylor series for a function at a given center.

    (b) Write a Taylor series in Σ-notation, and interpret sums written in that notation.

    (c) Know certain Taylor series centered at $b = 0$: e^x, $\sin(x)$, $\cos(x)$, and $\dfrac{1}{1-x}$.

25. **Operations on Taylor Series (Taylor Notes §5)**

    (a) Use known Taylor series to compute Taylor series for more complicated functions.

    (b) Apply addition, multiplication, differentiation, and integration to Taylor series.

    (c) Use Taylor series to compute higher derivatives without doing a lot of work.

    (d) Determine the intervals on which these derived Taylor series converge.