
A List of Topics for the Second Midterm

Here's a list of things you should be comfortable doing for the exam.

Old Stuff**1. Three-Dimensional Coordinate Systems (Chapter 12.1)**

- (a) Plot points in three dimensions.
- (b) Compute the distance between two points in \mathbf{R}^3 .
- (c) Recognize equations for cylinders and spheres.

2. Vectors (Chapter 12.2)

- (a) Recognize vectors written in a variety of forms.
- (b) Find a vector from one point to another.
- (c) Add, subtract, and scale vectors, either geometrically or algebraically.
- (d) Compute the length of a vector.

3. The Dot Product (Chapter 12.3)

- (a) Compute the dot product between two vectors.
- (b) Determine when two vectors are parallel or perpendicular.
- (c) Find the angle between two vectors.
- (d) Compute $\text{proj}_{\mathbf{a}}(\mathbf{b})$ and $\text{comp}_{\mathbf{a}}(\mathbf{b})$.

4. The Cross Product (Chapter 12.4)

- (a) Compute the cross product of two vectors in \mathbf{R}^3 .
- (b) Understand the connection between the directions of \mathbf{a} , \mathbf{b} , and $\mathbf{a} \times \mathbf{b}$.
- (c) Find the area of a triangle or parallelogram using the cross product.

5. Lines & Planes (Chapter 12.5)

- (a) Find the equation for a line given a point and a direction vector.
- (b) Find the equation for a plane given a point and a normal vector.
- (c) Solve all sorts of problems involving lines & planes, including but not limited to:
 - Check whether two lines are parallel, intersecting, or skew.
 - Find the intersection of two planes.
 - Find the intersection of a line and a plane.
 - Find a plane through three points.
 - Find a plane through a point and a line.
 - Find the distance from a point to a plane.
 - Find the angle between two planes.

6. Quadric Surfaces (Chapter 12.6)

- (a) Complete the square to write the equation for a quadric surface in standard form.
- (b) Recognize various quadric surfaces from their equations.
- (c) Determine the shape of a quadric surface by drawing its traces.
- (d) Find the intersection(s) of a line with a quadric surface.

7. Vector Functions and Space Curves (Chapter 13.1 & 10.1)

- (a) Compute limits of vector functions.
- (b) Sketch the space curve of a vector function.
- (c) Check whether the space curves of two vector functions intersect, and if so where.
- (d) Locate the intersection of a space curve and a quadric surface.
- (e) Find a vector function to represent the intersection of two surfaces.

8. Derivatives and Integrals of Vector Function (Chapter 13.2 & 10.2)

- (a) Take the derivative of a vector function.
- (b) Find the tangent vector to a space curve at a given point.
- (c) Compute antiderivatives of vector functions.
- (d) Determine the arc length of a two-dimensional parametric curve.

9. Polar Coordinates (Chapter 10.3)

- (a) Convert points and equations between polar form and Cartesian form.
- (b) Find tangent lines to polar functions.
- (c) (Roughly) sketch polar functions.

New Stuff

10. Arc Length and Curvature (Chapter 13.3)

- (a) Compute arc length for vector functions in three or more dimensions.
- (b) Find κ , \mathbf{T} , \mathbf{N} , and \mathbf{B} for a given vector function.
- (c) Use \mathbf{T} and \mathbf{B} to find normal and osculating planes to a space curve.

11. Velocity & Acceleration (Chapter 13.4)

- (a) Compute velocity and acceleration vectors for an object using its position vector.
- (b) Integrate to find the position vector using the acceleration vector.
- (c) Apply the equation $\mathbf{F} = m\mathbf{a}$.
- (d) Decompose an acceleration vector into its normal and tangential components.

12. Functions of Several Variables (Chapter 14.1)

- (a) Find the domain of a function of two or more variables.
- (b) Analyze the level curves of a function of several variables.
- (c) Sketch a function of two variables, when possible.

13. Partial Derivatives (Chapter 14.3)

- (a) Compute the partial derivatives of a function of two or more variables.
- (b) Interpret those partial derivatives as slopes.
- (c) Find tangent vectors to a multivariable function at certain points.
- (d) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
- (e) Compute higher derivatives, and apply Clairaut's theorem.

14. Tangent Planes and Linear Approximations (Chapter 14.4)

- (a) Find the tangent plane to a function of two variables at a given point.
- (b) Linearize a function at some point, and use it to approximate things.
- (c) Use differentials to estimate change in a multivariable function.

15. Optimization (Chapter 14.7)

- (a) Compute the critical points of a function of two variables.
- (b) Distinguish between local minima, local maxima, and saddlepoints.
- (c) Recognize local extrema conceptually, or using level curves.
- (d) Find the absolute maximum and minimum values of $f(x, y)$ over some domain.

16. Double Integrals over Rectangles (Chapter 15.1)

- (a) Estimate the volume under a surface using double Riemann sums.
- (b) Interpret double integrals as volumes, and compute familiar ones.
- (c) Find the average value of a function, based on this double integral.

17. Iterated Integrals (Chapter 15.2)

- (a) Use iterated integrals to compute the exact volume over a rectangular region.
- (b) Use Fubini's theorem to reverse the order of integration when necessary.

18. Double Integrals over General Regions (Chapter 15.3)

- (a) Find the volume under a surface over a non-rectangular region.
- (b) Set up iterated integrals based on a description of the region.
- (c) Reverse the order of integration, when necessary.

19. Double Integrals in Polar Coordinates (Chapter 15.4)

- (a) Recognize a double integral that would be more easily solved in polar coordinates.
- (b) Rewrite a double integral so that this computation is possible.
- (c) Set up a polar integral based on a description of the region.