Math 124 F - Autumn 2015
Midterm Exam Number Two
November 24, 2015

Name: ______________________________ Student ID no. : ____________________

Signature: ______________________________ Section: ______________

1	12	
2	12	
3	12	
4	12	
5	12	
Total	60	

- This exam consists of FIVE problems on SIX pages, including this cover sheet.
- Show all work for full credit.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you run out of room, write on the back of the page, but *indicate that you have done so*!
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You may use a *scientific calculator*. Calculators with graphing, differentiation, integration, or algebraic capabilities are not allowed.
- You have 80 minutes to complete the exam.

1. **[4 points per part]** Compute $\dfrac{dy}{dx}$.

(a) $y = \ln(2^x + x^2)$

$$\boxed{y' = \frac{2^x \ln(2) + 2x}{2^x + x^2}}$$

(b) $y = 2x - \dfrac{3}{(\sec(x) + 5)^2} = 2x - 3(\sec(x) + 5)^{-2}$

$$\boxed{y' = 2 + \frac{6\sec(x)\tan(x)}{(\sec(x) + 5)^3}}$$

(c) $y = (x + 5)^{\cos(x)}$

$$\Big\downarrow \ln(\ )$$

$$\ln(y) = \ln\left((x+5)^{\cos(x)}\right)$$

$$\ln(y) = \cos(x)\ln(x+5)$$

$$\Big\downarrow \frac{d}{dx}$$

$$\frac{y'}{y} = -\sin(x)\ln(x+5) + \frac{\cos(x)}{x+5}$$

$$y' = y\left(-\sin(x)\ln(x+5) + \frac{\cos(x)}{x+5}\right)$$

$$\boxed{y' = (x+5)^{\cos(x)}\left(-\sin(x)\ln(x+5) + \frac{\cos(x)}{x+5}\right)}$$

2. **[12 points]** Consider the following parametric curve on the domain $t \geq -1$:

$$x(t) = e^{2t} + 2 \qquad y(t) = \sqrt{t+1}$$

Find the equation of a tangent line to this curve that passes through the point $(2, 0)$.

Let $\left(e^{2t}+2, \sqrt{t+1}\right)$ be the point of tangency. Then the slope is: $\dfrac{y'(t)}{x'(t)} = \dfrac{\frac{1}{2\sqrt{t+1}}}{2e^{2t}} = \dfrac{1}{4e^{2t}\sqrt{t+1}}$

but the line passes through $\left(e^{2t}+2, \sqrt{t+1}\right)$ and $(2,0)$, so it's also: $\dfrac{\sqrt{t+1}}{e^{2t}+2-2}$

So these are equal:

$$\frac{1}{4\cancel{e^{2t}}\sqrt{t+1}} = \frac{\sqrt{t+1}}{\cancel{e^{2t}}}, \text{ so } t+1 = \frac{1}{4}, \text{ so } t = \frac{-3}{4}.$$

That makes the slope $\dfrac{\sqrt{\frac{-3}{4}+1}}{e^{\frac{-3}{2}}} = \dfrac{1}{2}\sqrt{e^{3}}$

So the tangent line is $\boxed{y = \frac{1}{2}\sqrt{e^{3}}\,(x-2)}$

3. Consider the implicit equation

$$\arctan(2-x)y + 2x = y^2 - 5.$$

(a) **[9 points]** Find the equation of the line tangent to this curve at the point $(2, 3)$.

$\frac{d}{dx}$

$$\frac{-y}{1+(2-x)^2} + \arctan(2-x)y' + 2 = 2yy'$$

$x=2,\ y=3$

$$\frac{-3}{1+0^2} + \arctan(0)y' + 2 = 6y'$$

$$y' = \frac{-1}{6}$$

So the line is $\boxed{y = \frac{-1}{6}(x-2)+3}$

(b) **[3 points]** Use the tangent line approximation to estimate the x-coordinate of the point on the curve near $(2, 3)$ with y-coordinate 3.04.

$$3.04 = \frac{-1}{6}(x-2)+3$$

$$\boxed{x = 1.76}$$

4. **[12 points]** Let $f(x) = 13\ln(x) + \frac{15}{x} - 2x$.

Find the absolute minimum and maximum values of $f(x)$ over the interval $[1, 4]$.

① Continuous! (b/c each term is continuous for $x>0$.)

Closed!

② $f'(x) = \frac{13}{x} - \frac{15}{x^2} - 2 = \frac{-2x^2+13x-15}{x^2} = \frac{(3-2x)(x-5)}{x^2}$ → zero @ $x=\frac{3}{2}$ & 5

→ undefined @ $x=0$

On interval $[1,4]$, only critical number is $\frac{3}{2}$.

③ $f(1) = 13$

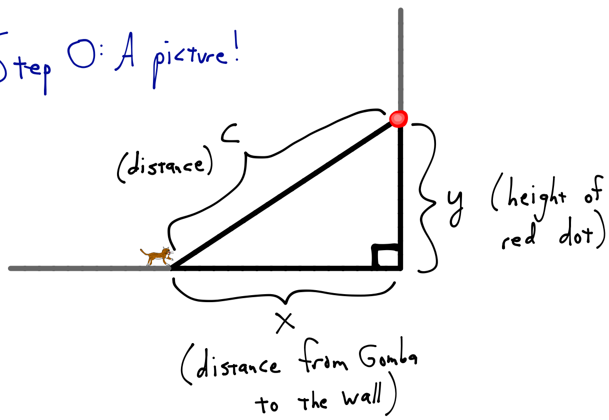
$$\boxed{\begin{aligned} f\left(\tfrac{3}{2}\right) &= 12.271 \leftarrow \text{absolute min} \\ f(4) &= 13.772 \leftarrow \text{absolute max} \end{aligned}}$$

5. **[12 points]**

Gomba is running towards a vertical wall at a speed of 11 feet per second, while a bright red dot is moving up the wall at a speed of 5 feet per second. At the moment when Gomba is 5.6 feet from the wall and the dot is 9 feet from the ground, what is the rate of change of the distance between Gomba and the dot? Is this distance increasing or decreasing?

(Ignore Gomba's height; he is crouched very low to the ground.)

Step 0: A picture!



Step 1: We know $x = 5.6$ ft, $y = 9$ ft,

$\frac{dx}{dt} = -11$ ft/sec, $\frac{dy}{dt} = 5$ ft/sec

We want $\frac{dc}{dt}$.

Step 2: $x^2 + y^2 = c^2$

Step 3: $\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}c\frac{dc}{dt}$.

Step 4: $(5.6)(-11) + (9)(5) = c\frac{dc}{dt}$

$$\frac{-16.6}{c} = \frac{dc}{dt}$$

But what's c?

Well, $5.6^2 + 9^2 = c^2$, so

$$c = 10.6$$

$$\frac{dc}{dt} = \frac{-16.6}{10.6} = \boxed{-1.566 \text{ ft/sec}}$$

It's $\boxed{\text{decreasing.}}$