Math 124 F - Autumn 2015
Midterm Exam Number One
October 27, 2015

Name: ______________________ Student ID no. : ______________

Signature: ______________________ Section: __________

1	15	
2	9	
3	7	
4	16	
5	13	
Total	60	

- This exam consists of FIVE problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you run out of room, write on the back of the page, but *indicate that you have done so*!
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You may use a *scientific calculator*. Calculators with graphing, differentiation, integration, or algebraic capabilities are not allowed.
- You have 80 minutes to complete the exam.

∘

1. **[5 points per part]** Compute each limit. You may use any techniques you know.

   If a limit does not exist or is infinite, say so, and explain.

   (a) $\displaystyle \lim_{x\to 8} \frac{\sqrt{x-4}+2}{x-3} = \frac{\sqrt{8-4}+2}{8-3} = \boxed{\frac{4}{5}}$

   Direct substitution:

   (b) $\displaystyle \lim_{t\to 0} \frac{\sin(at)-bt+ct^2}{t} = \lim_{t\to 0} \frac{\sin(at)}{t} + \lim_{t\to 0} \frac{-bt+ct^2}{t}$

   $\displaystyle = \lim_{t\to 0} \frac{a\sin(at)}{at} + \lim_{t\to 0}(-b+ct)$

   $= \boxed{a-b}$

   (c) $\displaystyle \lim_{x\to\infty} \sin\left(\frac{\pi x+6}{\sqrt{4x^2+2x}+2x}\right) = \sin\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$

   $\displaystyle \lim_{x\to\infty} \frac{\pi x+6}{\sqrt{4x^2+2x}+2x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

   $\displaystyle = \lim_{x\to\infty} \frac{\pi + \underbrace{\frac{6}{x}}_{\to 0}}{\sqrt{4+\underbrace{\frac{2}{x}}_{\to 0}}+2} = \frac{\pi}{4}$

2. **[9 points]** Consider the curve $y = \sqrt{x} + 4x$.

Give the equation for a tangent line to this curve which is parallel to the line $y = 5x + 4$.

$$y' = \frac{1}{2\sqrt{x}} + 4 = 5$$

slope $= 5$

$$\frac{1}{2\sqrt{x}} = 1$$

$$\sqrt{x} = \frac{1}{2}$$

$$x = \frac{1}{4}$$

Point of tangency:

$$\left(\frac{1}{4}, \sqrt{\frac{1}{4}} + 4\left(\frac{1}{4}\right)\right)$$

$$= \left(\frac{1}{4}, \frac{3}{2}\right)$$

Point-slope formula:

$$\boxed{y = 5\left(x - \frac{1}{4}\right) + \frac{3}{2}}$$

3. **[7 points]** Consider the function $f(x) = \sec(x) - xe^x$. Compute $f''(x)$.

$$f'(x) = \sec(x)\tan(x) - (e^x + xe^x)$$

$$f''(x) = \sec(x)\tan^2(x) + \sec^3(x) - (e^x + e^x + xe^x)$$

$$\boxed{f''(x) = \sec(x)(\tan^2(x) + \sec^2(x)) - (2+x)e^x}$$

4. Consider the following multipart function:

$$f(x) = \begin{cases} ae^x + b\cos(x) + 3x & \text{if } x \le 0 \\ \dfrac{x+5}{x^2 - 2x + 1} & \text{if } x > 0 \end{cases}$$

(a) **[10 points]** Determine constants a and b so that $f(x)$ is differentiable at $x = 0$.

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left(ae^x + b\cos(x) + 3x \right) = a + b$$

$a + b = 5$ (or else $f(x)$ is not continuous @ $x = 0$.)

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(\frac{x+5}{x^2 - 2x + 1} \right) = 5$$

Also: $$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} \left(ae^x - b\sin(x) + 3 \right) = a + 3$$

$a + 3 = 11$

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} \frac{(1)(x^2 - 2x + 1) - (2x - 2)(x + 5)}{(x^2 - 2x + 1)^2} = 11$$

Solve:
$a + b = 5$
$a + 3 = 11$

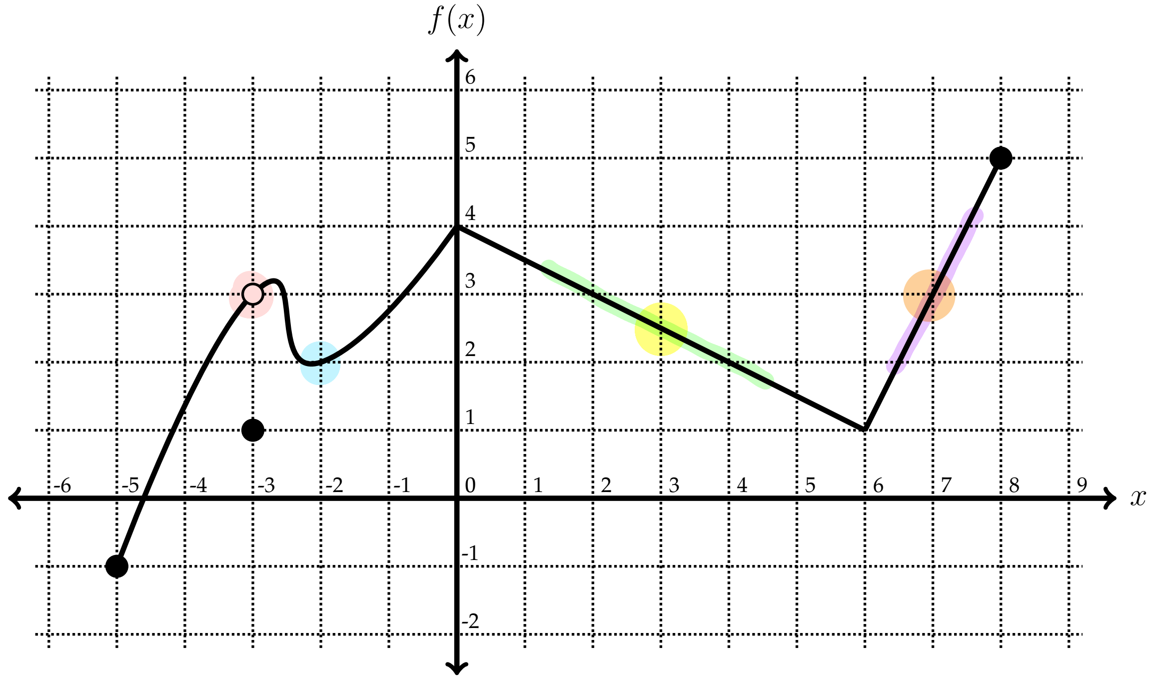
$$\boxed{\begin{aligned} a &= 8 \\ b &= -3 \end{aligned}}$$

(b) **[6 points]** Find all horizontal asymptotes of $f(x)$.

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \left(\underbrace{ae^x}_{0} + \underbrace{b\cos(x)}_{\text{small}} + \underbrace{3x}_{-\infty} \right) = -\infty,$$ no H.A. to the left.

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left(\frac{x + 5}{x^2 - 2x + 1} \right) = 0,$$ so $\boxed{y = 0}$ is a H.A.

5. The graph of $f(x)$ is shown below.



Cool graph, right? Use it to answer the following questions.

(a) **[3 points]** Compute $\lim_{x\to-3}[f(x)\cdot f(x+1)] = \boxed{6}$

$\lim_{x\to-3} f(x) \to 3$, $\lim_{x\to-3} f(x+1) \to 2$

(b) **[3 points]** For what constant c does $\lim_{x\to3}\dfrac{f(x)-c}{x-3}$ exist?

$$\lim_{x\to3}\frac{f(x)-c}{x-3} \qquad f(x)-c \to 2.5-c, \quad x-3\to 0$$

so $2.5-c=0$ or the limit DNE.

$$\boxed{c=2.5}$$

(c) **[3 points]** Compute the limit from part (b), using the value of c you chose.

Wait, $\displaystyle\lim_{x\to3}\frac{f(x)-2.5}{x-3} = f'(3) = \boxed{\frac{-1}{2}}$

(d) **[4 points]** Let $g(x) = \dfrac{f'(x)}{f(x)}$. What is $g'(7)$?

Quotient rule: $\displaystyle g'(7) = \frac{f''(7)f(7)-f'(7)f'(7)}{[f(7)]^2}$

$$= \frac{0(3)-2^2}{3^2} = \boxed{\frac{-4}{9}}$$

$f(7)=3$

$f'(7)=2$

$f''(7)=0$ (constant slope, so f' is constant around $x=7$)