
A List of Topics for the Final

Here's a fairly comprehensive list of things you should be comfortable doing for the final exam.

Really Old Stuff

1. Unit conversion and rates of change.
2. Coordinate systems.
 - (a) Plotting things in a coordinate system.
 - (b) Using the distance formula.
3. Equations for lines and circles.
 - (a) Finding intersections of curves.
 - (b) Writing equations for circles and semicircles.
4. Linear modeling.
 - (a) Finding an equation for a line given various pieces of information. Finding the shortest distance from a line to a point not on that line.
 - (b) Using linear equations for real-world problems with constant rates of change.
 - (c) Finding parametric equations for linear motion.
5. Functions and graphing.
 - (a) Graphing a function, and analyzing a function based on its graph.
 - (b) Evaluating functions, and solving equations like $f(2x + 3) = x$.
6. Graphical analysis.
 - (a) Determining the domain and range of a function, visually or algebraically, and using the vertical line test.
 - (b) Graphing, constructing, and solving multipart functions.

Old Stuff

7. Quadratic modeling.
 - (a) Graphing quadratic functions and converting to vertex form.
 - (b) Finding the minimum and/or maximum values of quadratic functions over certain ranges.
 - (c) Finding a formula for a quadratic function through a given set of points, and/or with a given vertex or line of symmetry.
 - (d) Constructing a quadratic to find the minimum and maximum values of certain real-world functions.

8. Functional composition.

- (a) Giving a formula for $f(g(x))$ based on the formulas for $f(x)$ and $g(x)$.
- (b) Determining the domain and range of the composition of functions.
- (c) Computing $f(g(x))$ when f and/or g are multipart functions.
- (d) Computing “fixed points” of a function $f(x)$. That is, finding solutions to the equation $f(x) = x$.

9. Inverse functions.

- (a) Computing the inverse of a function algebraically, and drawing the inverse of a function graphically.
- (b) Determining whether a function is one-to-one, both algebraically and graphically.
- (c) For certain functions that *aren't* one-to-one (e.g. parabolas), knowing how to break those functions down into smaller parts, and finding inverses for each of those pieces.

10. Exponential functions.

- (a) Computing and manipulating exponential functions.
- (b) Knowing the various rules of exponents.
- (c) Converting exponential functions into “standard exponential form”.

11. Exponential modeling.

- (a) Finding an exponential function to match real-world data.

12. Logarithmic functions.

- (a) Relating logarithms to exponential functions, and using them to solve exponential equations.
- (b) Manipulating said equations by the properties of logarithmic functions.
- (c) Graphing logarithmic functions.

13. Graphical transformations.

- (a) Manipulating an equation algebraically in order to translate, reflect, and/or dilate its graph.
- (b) Drawing a graph based on an equation, after it has had the above transformations applied.

14. Rational functions.

- (a) Graphing linear-to-linear rational functions and computing their asymptotes.
- (b) Finding a linear-to-linear rational function based on data points and/or asymptotes.
- (c) Using linear-to-linear rational functions to model real-world problems.

15. Measuring angles.

- (a) Converting between radians and degrees.

- (b) Finding the lengths of circular arcs, and approximating the lengths of chords when the subtended angle is small.
- (c) Finding areas of sectors and other reasonable shapes involving circles.

New Stuff

16. Circular motion.

- (a) Using linear and angular speeds to describe movement on a circle.
- (b) Using equations to relate angular speed, linear speed, arc length, subtended angle, and the radius of a circle.
- (c) Solving problems involving wheels, belts, and axles.

17. Circular functions.

- (a) Using $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, that kind of stuff.
- (b) Solving problems involving triangles, angles, and lengths.
- (c) Solving all sorts of questions involving people moving around a circle, including their coordinates (parametric equations), equations for tangent lines, distances, and times and locations where they pass each other.

18. Trigonometric functions.

- (a) Graphing the trigonometric functions introduced in the previous chapter.
- (b) Applying trigonometric identities for various purposes (especially in the next few chapters).

19. Sinusoidal modeling.

- (a) Relating the equation for a sinusoidal function to its amplitude, period, phase shift, and mean.
- (b) Graphing a sinusoidal function based on its equation, or finding the equation based on the graph.
- (c) Creating a sinusoidal function to meet the constraints of a word problem, and using it to answer more questions about something with sinusoidal behavior.

20. Inverse trigonometric functions.

- (a) Understanding the definitions and graphs of $\sin^{-1}(x)$, $\cos^{-1}(x)$, and $\tan^{-1}(x)$, as well as their domains and ranges.
- (b) Using inverse trigonometric functions to solve *more* questions involving lengths, triangles, and angles, especially ones that require you to solve for a certain angle.
- (c) Solving sinusoidal modeling problems that ask when a certain value is reached, or how long the function spends above or below a certain value.
- (d) Understanding how to find the principal and symmetry solutions to sinusoidal equations, and in particular how this relates to answering questions in the note above.

Some Useful Equations

- The distance d between points (x_1, y_1) and (x_2, y_2) : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- A line through points (x_1, y_1) and (x_2, y_2) : $y = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1) + y_1$
- A line through the point (x_1, y_1) with slope m : $y = m(x - x_1) + y_1$
- A line with y -intercept b and slope m : $y = mx + b$
- A circle with center (x_0, y_0) and radius r : $(x - x_0)^2 + (y - y_0)^2 = r^2$
- The parametric equations for uniform linear motion from (x_0, y_0) to (x_1, y_1) in Δt units of time, where $\Delta x = x_1 - x_0$, and $\Delta y = y_1 - y_0$:

$$x = x_0 + \frac{\Delta x}{\Delta t}t \quad y = y_0 + \frac{\Delta y}{\Delta t}t$$

- An upper semicircle with center (x_0, y_0) and radius r : $y = y_0 + \sqrt{r^2 - (x - x_0)^2}$
- A lower semicircle with center (x_0, y_0) and radius r : $y = y_0 - \sqrt{r^2 - (x - x_0)^2}$
- A quadratic, with vertex (h, k) and scaling factor a : $y = a(x - h)^2 + k$
- Converting to vertex form from $y = ax^2 + bx + c$: $h = \frac{-b}{2a} \quad k = c - \frac{b^2}{4a}$
- An exponential with starting value A_0 and annual growth factor b : $y = A_0b^x$
- Properties of exponential functions and logarithms:

$$b^x b^y = b^{x+y} \quad \frac{b^x}{b^y} = b^{x-y} \quad (b^x)^y = b^{xy}$$

$$(ab)^x = a^x b^x \quad b^{-x} = \frac{1}{b^x} \quad b^0 = 1$$

$$\ln(xy) = \ln(x) + \ln(y) \quad \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y) \quad \ln(x^y) = y \ln(x)$$

$$\log_b(x) = \frac{\ln(x)}{\ln(b)} \quad \ln(e^x) = x \quad e^{\ln(x)} = x$$

- A linear-to-linear rational function, asymptotes $y = a$ and $x = -d$: $y = \frac{ax + b}{x + d}$
- Length s of an arc subtended by angle θ (in rad.) in a circle of radius r : $s = \theta r$
- Area A of a sector subtended by angle θ (in rad.) in a circle of radius r : $A = \frac{1}{2}\theta r^2$
- Linear speed v and angular speed ω moving around a circle of radius r : $v = \omega r$

- Basic trig functions:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}} \quad \sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}} \quad \cot(\theta) = \frac{\text{adjacent}}{\text{opposite}}$$

- Parametric equations for uniform circular motion with starting angle θ_0 and angular speed ω around a circle of radius r centered at (x_0, y_0) . The \pm s are $+$ s if the motion is counterclockwise, and $-$ s if clockwise:

$$x = r \cos(\theta_0 \pm \omega t) + x_0 \quad y = r \sin(\theta_0 \pm \omega t) + y_0$$

- General formula for a sinusoidal function with amplitude A , period B , phase shift C and average value D :

$$f(x) = A \sin\left(\frac{2\pi}{B}(x - C)\right) + D$$

- Symmetry solution s for a sinusoidal function with period B and phase shift C , if the principal solution is p : $s = 2C + B/2 - p$