DFEP #1: Monday, January 12th.

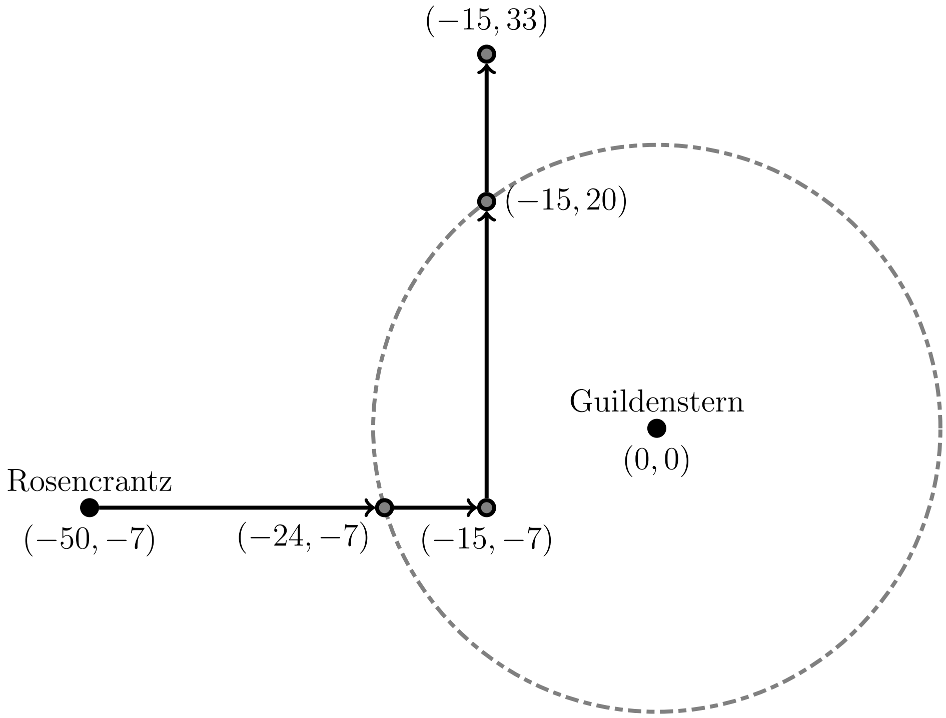
Rosencrantz is standing 7 units south and 50 units west of Guildenstern on a very large sheet of graph paper. Rosencrantz walks 35 units east at a speed of 4 units per second, then immediately turns and walks 40 units north at a speed of 3 units per second.

During this process, how much time does Rosencrantz spend within 25 units of Guildenstern?

DFEP #1 Solution:

Here's a picture. Rosencrantz walks along the line $y = -7$ at a speed of 4 units/second, and then along the line $x = -15$ at a speed of 3 units/second. The intersection points are found by plugging those lines (individually) into the equation of the circle.

He spends $9/4 = 2.25$ seconds walking east inside that circle, and $27/3 = 9$ seconds walking north inside the circle, for a total of 11.25 seconds.



DFEP #2: Wednesday, January 14th.

Jayne stands 24 miles west and 10 miles south of the **westernmost** point of a circular forest with radius 13 miles. She begins walking in a straight line towards the **easternmost** point of the forest. She walks through the forest at a speed of 1 mile/hour.

At the time when Jayne is closest to the center of the forest, how long has she been inside the forest?

DFEP #2 Solution:

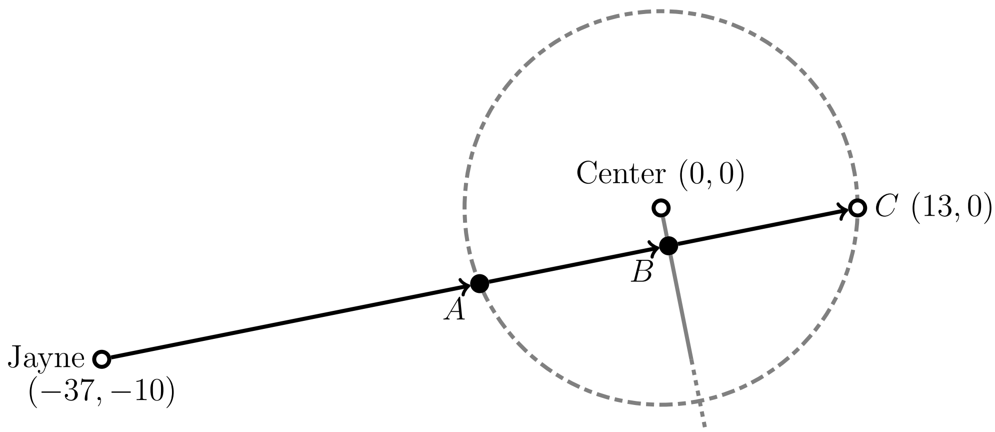
Here's a picture. Jayne walks from $(-37, -10)$ to $(13, 0)$ at a speed of 1 mile per hour. We want the time it takes her to walk from A to B. To find A's coordinates, we take the equation of the line she walks along:

$$y = \frac{10}{50}(x - 13) + 0$$

and plug that into the equation of the circle, $x^2 + y^2 = 13^2$. Solving for x and y gives the point $(-12, -5)$.

To find B's coordinates, we use the fact that the line between B and the center is perpendicular to the previous line, so its equation is $y = -50x/10$.

Once you solve for A and B's coordinates, you plug them into the distance formula and divide by her speed (which is just 1 mph). The time is about 12.7475 hours, or (if you prefer) 12 hours, 44 minutes, and 51 seconds.



DFEP #3: Friday, January 16th.

Niamh stands at $(5, 2)$ and begins walking to $(1, 7)$, reaching it in 3 seconds.

Meanwhile, Tom heads from $(4, -3)$ towards $(5.8, 5)$ at a speed of 2 units per second.

Find the distance between Niamh and Tom t seconds after they both start walking.

DFEP #3 Solution:

We'll want parametric equations for both people. For Niamh, this is straightforward:

$$x = 5 + \frac{-4}{3}t \qquad\qquad y = 2 + \frac{5}{3}t$$

For Tom, we need to know how long it takes him to get from $(4, -3)$ to $(5.8, 5)$. Using the distance formula, those two points are $\sqrt{(4-5.8)^2 + (-3-5)^2} = 8.2$ units apart. He walks at a speed of 2 units per second, so it takes him 4.1 seconds. So Tom's parametric equations are:

$$x = 4 + \frac{1.8}{4.1}t \qquad\qquad y = -3 + \frac{8}{4.1}t$$

(Note: The above formula has been edited to remove an extra minus sign. The version shown in class was incorrect.)

Plugging these into the distance formula, we have the distance between them as

$$d = \sqrt{\left(\left[5 + \frac{-4}{3}t\right] - \left[4 + \frac{1.8}{4.1}t\right]\right)^2 + \left(\left[2 + \frac{5}{3}t\right] - \left[-3 + \frac{8}{4.1}t\right]\right)^2}$$

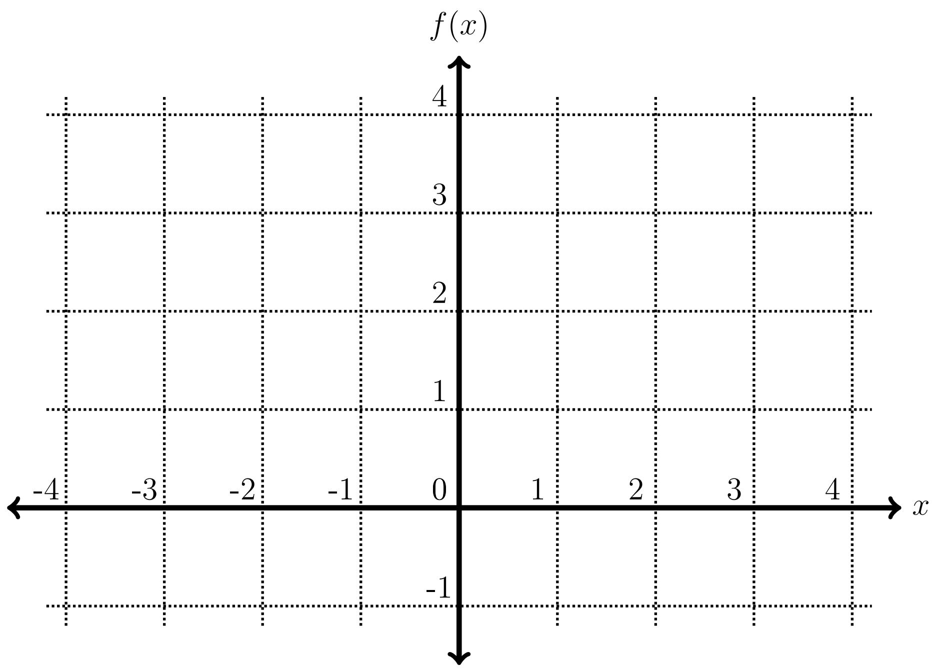
which you could simplify if you'd like, but that's a fine answer on an exam.

DFEP #4: Wednesday, January 21st.

Consider the function

$$f(x) = \begin{cases} 2 & \text{if } -3 < x < -1 \\ 1 - x & \text{if } -1 \le x < 1 \\ 1 + \sqrt{4 - (x-3)^2} & \text{if } 1 \le x \le 3 \end{cases}$$

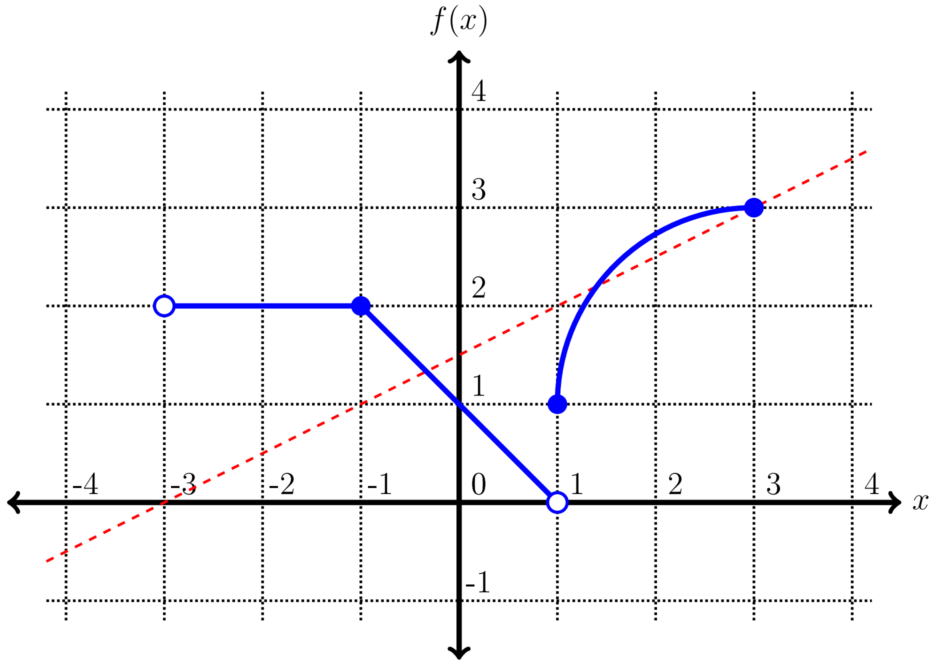
(a) Sketch a carefully-labeled graph of $f(x)$.



(b) Find all solutions to the equation $f(x) = (x+3)/2$.

DFEP #4 Solution:

(a) The graph consists of two line segments and one quarter circle, shown in blue below:



(b) To solve the equation, we set $(x+3)/2$ equal to all three pieces of $f(x)$:

Solving $(x+3)/2 = 2$ tells us that $x = 1$, which does not satisfy the condition $-3 < x < -1$. So that isn't a solution.

Solving $(x+3)/2 = 1-x$ tells us that $x = -1/3$, which **does** satisfy the inequality $-1 \leq x < 1$, so $x = -1/3$ is one solution.

Finally, solving $(x+3)/2 = 1 + \sqrt{4-(x-3)^2}$, after the quadratic formula, says that $x = 1.4$ or $x = 3$, both of which satisfy the inequality $1 \leq x \leq 3$, so in total we have three solutions: $x = -1/3$, $x = 1.4$, and $x = 3$.

Notice that this answer makes sense visually: if we plot the equation $y = (x+3)/2$, we get the dotted red line shown above, which does seem to intersect the blue curve at $x = -1/3$, $x = 1.4$, and $x = 3$.

DFEP #5: Friday, January 23rd.

Timon bikes north from his house at a speed of 0.4 miles per minute for 25 minutes. Then, he turns and bikes at a constant speed straight toward a point 6 miles west and 2 miles north of his house, reaching it in 20 minutes.

(a) Give a multipart function for Timon's distance from his house, in miles, t minutes after he begins his journey.

(b) Find all times on Timon's journey when he is exactly 7.5 miles from home.

DFEP #5 Solution:

Let's set our coordinates so that Timon's house is at $(0, 0)$. If he bikes north at a speed of 0.4 miles per minute for 25 minutes, then during the first 25 minutes his distance from home is simply $0.4t$.

From 25 minutes to 45 minutes, he bikes from $(0, 10)$ to $(-6, 2)$ in 20 minutes, so $\Delta x = -6$, $\Delta y = -8$, and $\Delta t = 20$. But because there's a 25-minute delay before he starts this part of his journey, the parametric equations during this time period are:

$$x = 0 + \frac{-6}{20}(t - 25) \qquad y = 10 + \frac{-8}{20}(t - 25)$$

The distance from (x, y) to home during this time period is simply $\sqrt{x^2 + y^2}$, where x and y are given by the above formulae.

So, putting this all together, his distance from home is given by the multipart function

$$d(t) = \begin{cases} 0.4t & \text{if } 0 \leq t \leq 25 \\ \sqrt{\left(\frac{-6}{20}(t-25)\right)^2 + \left(10 + \frac{-8}{20}(t-25)\right)^2} & \text{if } 25 \leq t \leq 45 \end{cases}$$

DFEP #6: Monday, January 26th.

You have decided to begin selling chairs, and the profit you make from selling chairs for $\$x$ each is a quadratic function of x: if you charge \$20 per chair, you'll make a total profit of \$76. If you charge \$100 per chair, you'll make a total profit of \$396.

Suppose you know that you can maximize the profit by selling chairs for \$80 each. What will the profit be?

DFEP #6 Solution:

Let $f(x)$ be the profit (in dollars) after charging \$$x$ per chair. We have been told that $f(x)$ is a quadratic function satisfying three criteria: $f(20) = 76$, $f(100) = 396$, and the vertex of the parabola is at $h = 80$. We want to know what k is. We'll write the quadratic in vertex form, so $f(x) = a(x - h)^2 + k$. Substituting $h = 80$ into the first two facts tells us:

$76 = a(20 - 80)^2 + k$
$396 = a(100 - 80)^2 + k$

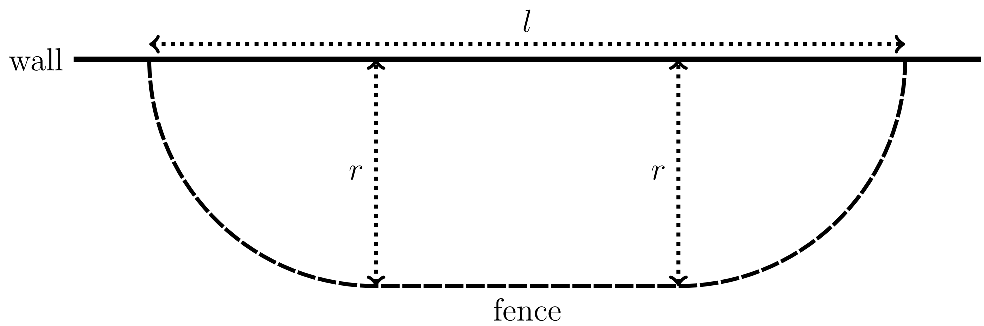
Subtracting the first equation from the second yields $320 = -3200a$, so $a = -0.1$. Therefore $396 = -0.1(100 - 80)^2 + k$, so $k = 436$. So the maximum possible profit is \$436.

DFEP #7: Friday, January 30th.

You want to build a fence along a wall. The fence should consist of two quarter-circular arcs connected by a straight line, as shown below.

Suppose l is the total length of the wall between the two ends of the fence, and r is the radius of the arcs. It is okay for the arcs or the straight line to have length zero.

If you have 100 meters of fencing to work with, what should l and r be in order to maximize the area enclosed by the fence?



DFEP #7 Solution:

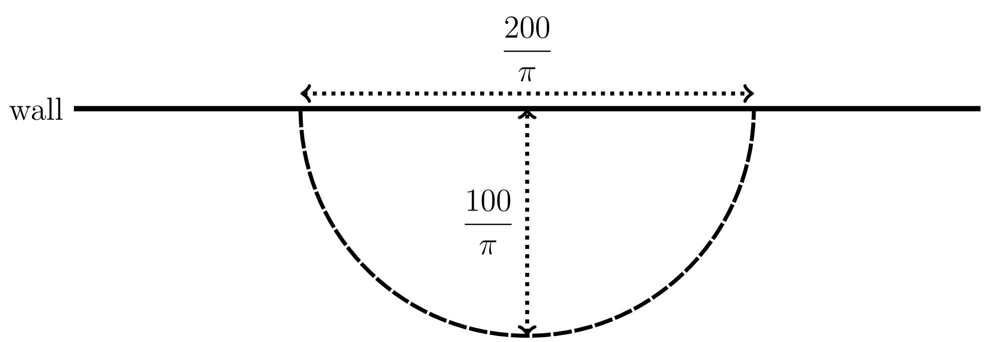
The fenced-off area consists of two quarter-circles (with total area $\pi r^2/2$) and one rectangle (with width r and length $l - 2r$), so the total area is $A = \pi r^2/2 + r(l - 2r)$.

Unfortunately, this formula has more than one variable, so let's figure out how long the fence is: the circular pieces are πr meters long (in total, since it's half the circumference of a circle), and the straight line is $l-2r$ meters long, so the total length is $\pi r + l - 2r = 100$. Therefore, $l = 100 + 2r - \pi r$. Plugging that into the area equation gives

$$A = \frac{1}{2}\pi r^2 + r(100 + 2r - \pi r - 2r) = -\frac{1}{2}\pi r^2 + 100r$$

This is a downward-pointing parabola, so the maximum occurs at $h = -100/\left(2\left(\frac{-1}{2}\pi\right)\right)$, which simplifies to $100/\pi$ meters. So $r = 100/\pi$, and $l = 100 + 2(100/\pi) - \pi(100/\pi) = 200/\pi$.

It's important to make sure that our answer is actually in the domain of the problem, though. Does it make sense for r to equal $100/\pi$ and l to equal $200/\pi$? Yeah, here's what that looks like:



Therefore, the best way to craft the fence is to make it a semicircle, so the straight line portion has length 0.

DFEP #8: Monday, February 2nd.

Find all possible linear functions $f(x)$ so that $f(f(x)) = 9x - 16$.

DFEP #8 Solution:

We want a linear function $f(x)$, so $f(x) = mx + b$. And we want $f(f(x))$ to equal $9x - 16$, so:

$$f(f(x)) = f(mx + b) = m(mx + b) + b = m^2x + mb + b$$

So $m^2x + mb + b = 9x - 16$. That means that $m^2 = 9$, and $mb + b = -16$.

Since $m^2 = 9$, we know that $m = 3$ or $m = -3$.

If $m = 3$, then $3b + b = -16$, so $b = -4$.

If $m = -3$, then $-3b + b = -16$, so $b = 8$.

So there are two solutions: $f(x) = 3x - 4$, or $f(x) = -3x + 8$.

DFEP #9: Wednesday, February 4th.

Find the inverse of $f(x) = \sqrt{x-1} + 5x$.

DFEP #9 Solution:

We begin by writing $y = \sqrt{x-1} + 5x$, which becomes $x = \sqrt{y-1} + 5y$, which we try to solve for y. First off, subtract $5y$ from both sides and square to get:

$$(x - 5y)^2 = y - 1$$

which simplifies to

$$25y^2 + (-10x - 1)y + x^2 + 1 = 0$$

which we solve via the quadratic formula to get

$$y = \frac{10x + 1 \pm \sqrt{(-10x-1)^2 - 4(25)(x^2+1)}}{50}$$

which simplifies to

$$y = \frac{10x + 1 \pm \sqrt{20x - 99}}{50}$$

Now, in the original function, we had $f(1) = 5$. So for the inverse, we need $f^{-1}(5) = 1$. If we plug in $x = 5$ to the equation above, we get $y = (51 \pm 1)/(50)$, which only equals 1 if that $\pm$ is a minus, not a plus. So the inverse is:

$$f^{-1}(x) = \frac{10x + 1 - \sqrt{20x - 99}}{50}$$

DFEP #10: Friday, February 6th.

Cindi's android factory produces robots at an exponential rate. In the year 2050, there were 25 robots produced. In the year 2100, there are 200 robots. How many robots will there be in the year 2719?

DFEP #10 Solution:

The number of robots multiplies by 8 every 50 years, so the exponential function looks like $f(x) = A_0(8)^{x/50}$. What's A_0? Well, let x be the number of years after the year 2000, so $x = 0$ corresponds to the year 2000. That's 50 years before 2050, when there were 25 robots, so the starting number of robots is 25/8. Which means that after x years there are $f(x) = (25/8)(8)^{x/50}$ robots. Plugging in $x = 719$ gives us 3.02889×10^{13} robots in the year 2719, so about 30 trillion.

DFEP #11: Monday, February 9th.

The country of Exponentia consists of two separate states, 4ida and 10essee, each of which has a population that grows at an exponential rate.

In the year 1990, 4ida has a population of 500,000 and 10essee has a population of 70,000.

Every ten years, the population of 4ida increases by a factor of 4, while the population of 10essee increases by a factor of 10. When will the states have equal populations?

DFEP #11 Solution.

Let's get formulas $f(t)$ and $g(t)$ for the populations in 4ida and 10essee, respectively, t years after 1990. We know that $f(0) = 500000$ and $g(0) = 70000$. Furthermore, we know that 4ida's population increases by a factor of 4 every 10 years, so $f(t) = 500000(4)^{t/10}$. Similarly, $g(t) = 70000(10)^{t/10}$.

We want to know when these populations are equal, so we solve:

$$500000(4)^{t/10} = 70000(10)^{t/10}$$

Take the natural log of both sides to get:

$$\ln(500000(4)^{t/10}) = \ln(70000(10)^{t/10})$$

which becomes

$$\ln(500000) + \ln(4^{t/10}) = \ln(70000) + \ln(10^{t/10})$$

which becomes

$$\ln(500000) + \frac{t}{10}\ln(4) = \ln(70000) + \frac{t}{10}\ln(10)$$

So

$$t = 10\frac{\ln(500000) - \ln(70000)}{\ln(10) - \ln(4)} \approx 21.45 \text{ years after 1990}$$

and therefore the populations are equal around the middle of 2011.

DFEP #12: Wednesday, February 11th.

A frittata and a galette are both removed from an oven at the same time at a temperature of 375° Fahrenheit. After t minutes out of the oven, their temperatures (in Fahrenheit) are given by exponential functions of t.

After 10 minutes, the frittata is 225°. After 30 minutes, the galette is 19° warmer than the frittata.

(a) Give a function $f(t)$ for the temperature of the frittata after t minutes.

(b) Give a function $g(t)$ for the temperature of the galette after t minutes.

(c) When is the galette's temperature (in Fahrenheit) twice as much as the frittata's?

DFEP #12 Solution:

(a) We want to find an equation $f(t) = A_0 b^t$, and we know that $f(0) = 375$ and $f(10) = 225$. We know:

$$375 = A_0 b^0 \qquad 225 = A_0 b^{10}$$

Therefore, $A_0 = 375$, and $b = \sqrt[10]{225/375} = \sqrt[10]{3/5}$.

Putting this all together yields $f(t) = 375\left(\sqrt[10]{3/5}\right)^t$.

(b) After 30 minutes, the galette is 19° warmer than the frittata, which translates to $g(30) = f(30) + 19 = 81 + 19 = 100$. Since the galette starts at the same temperature as the frittata, we know that $g(t) = 375b^t$ for some b, which gives:

$$100 = 375b^{30}$$

Solving this for b yields $b = \sqrt[30]{100/375} = \sqrt[30]{4/15}$, so $g(t) = 375\left(\sqrt[30]{4/15}\right)^t$

(c) We want to know when the galette's temperature is double the frittata's, so we want to solve $g(t) = 2f(t)$. That means

$$375\left(\sqrt[30]{4/15}\right)^t = 2 \cdot 375\left(\sqrt[10]{3/5}\right)^t$$

Taking the natural log of both sides and simplifying says that

$$\ln(375) + t\ln\left(\sqrt[30]{4/15}\right) = \ln(2) + \ln(375) + t\ln\left(\sqrt[10]{3/5}\right)$$

which means that

$$t = \frac{\ln(2)}{\frac{1}{30}\ln(4/15) - \frac{1}{10}\ln(3/5)} \approx 98.68$$

So the galette's temperature will be twice the frittata's after 98.68 minutes.

DFEP #13: Friday, February 13th.

Let $f(x) = 5x - x^2$.

(a) Compute $f(f(4))$.

(b) Restrict $f(x)$ to the domain $[2.5, \infty)$. Write a formula for $f^{-1}(x)$.

(c) Suppose $g(x)$ is the function formed by moving the graph of $f(x)$ two units to the right, then scaling horizontally by a factor of 4, then scaling vertically by a factor of 1/2, and finally moving two units down.

Write a formula for $g(x)$.

DFEP #13 Solution:

Once again, $f(x) = 5x - x^2$.

(a) $f(f(4)) = f(5(4) - 4^2) = f(20 - 16) = f(4) = 5(4) - 4^2 = 4$.

(b) We start by writing $x = 5y - y^2$ and solving for y in terms of x. Subtracting x from both sides gives the equation

$$-y^2 + 5y - x = 0$$

so the quadratic formula gives

$$y = \frac{-5 \pm \sqrt{5^2 - 4x}}{-2}$$

Is that $\pm$ a $+$ or a $-$? We restricted $f(x)$ to the domain $[2.5, \infty)$, which means that the range of the inverse should also be $[2.5, \infty)$. In order for this function to give outputs greater than 2.5, it should be a $-$, so

$$f^{-1}(x) = \frac{-5 - \sqrt{25 - 4x}}{-2}$$

(c) We start with $y = 5x - x^2$.
Then we replace x with $(x - 2)$ to get $y = 5(x - 2) - (x - 2)^2$.
Then we replace x with $x/4$ to get $y = 5((x/4) - 2) - ((x/4) - 2)^2$
Then we replace y with $y/(1/2)$ to get $y/(1/2) = 5((x/4) - 2) - ((x/4) - 2)^2$.
Finally, we replace y with $y - (-2)$ to get $(y - (-2))/(1/2) = 5((x/4) - 2) - ((x/4) - 2)^2$. This simplifies to

$$y = g(x) = \frac{1}{2}\left(5\left(\frac{x}{4} - 2\right) - \left(\frac{x}{4} - 2\right)^2\right) - 2$$

DFEP #14: Wednesday, February 18th.

Sybil's least favorite tree grows according to a linear-to-linear rational function of time. Right now, it's 8 feet tall. Five years ago, it was 4.5 feet tall. In the long run, its height will approach (but not reach) 15 feet. How tall will it be in 10 years?

DFEP #14 Solution:

We want a linear-to-linear rational function $f(x) = \dfrac{ax+b}{x+d}$, and we know $f(0) = 8$, $f(-5) = 4.5$, and $a = 15$. So:

$$8 = \frac{0+b}{0+d} \qquad 4.5 = \frac{-5a+b}{-5+d} \qquad a = 15$$

The first equation becomes $b = 8d$, and plugging $a = 15$ into the second gives

$$-22.5 + 4.5d = -75 + b.$$

So $-22.5 + 4.5d = -75 + 8d$, so $52.5 = 3.5d$, so $d = 15$, and $b = 120$. Therefore we get

$$f(x) = \frac{15x + 120}{x + 15}$$

In 10 years, we'll have a height of $f(10) = (150 + 120)/25 = 10.8$ feet.

DFEP #15: Friday, February 20th.

State the domain and range of a linear-to-linear rational function that passes through the points $(2, 0)$, $(-2, -16)$, and $(5, 1.5)$.

DFEP #15 Solution:

We want a function $f(x) = \dfrac{ax+b}{x+d}$ such that $f(2) = 0$, $f(-2) = -16$, and $f(5) = 1.5$. So we want to solve:

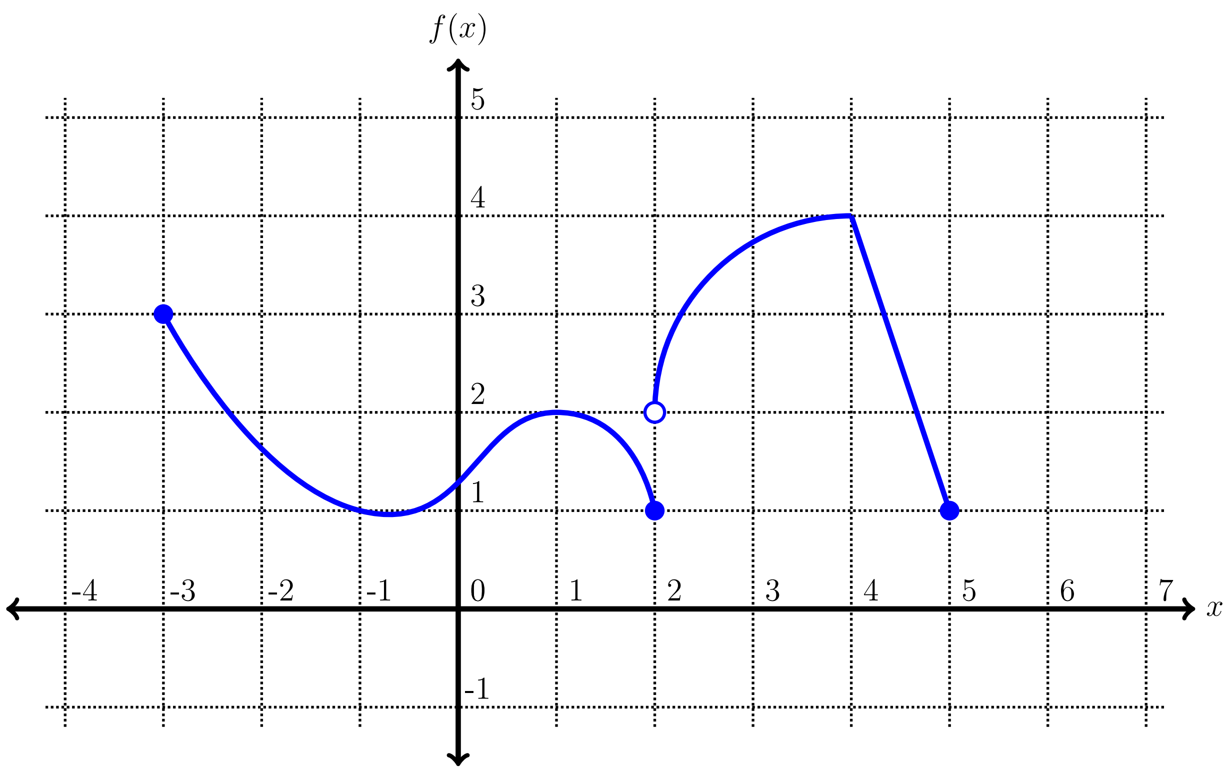
$$0 = \frac{2a+b}{2+d}$$
$$-16 = \frac{-2a+b}{-2+d}$$
$$1.5 = \frac{5a+b}{5+d}$$

Solving these three equations yields $a = 4$, $b = -8$, $d = 3$, so $f(x) = \dfrac{4x-8}{x+3}$.

The domain of $f(x)$ is $(-\infty, -3) \cup (-3, \infty)$, and the range is everything but the horizontal asymptote: $(-\infty, 4) \cup (4, \infty)$.

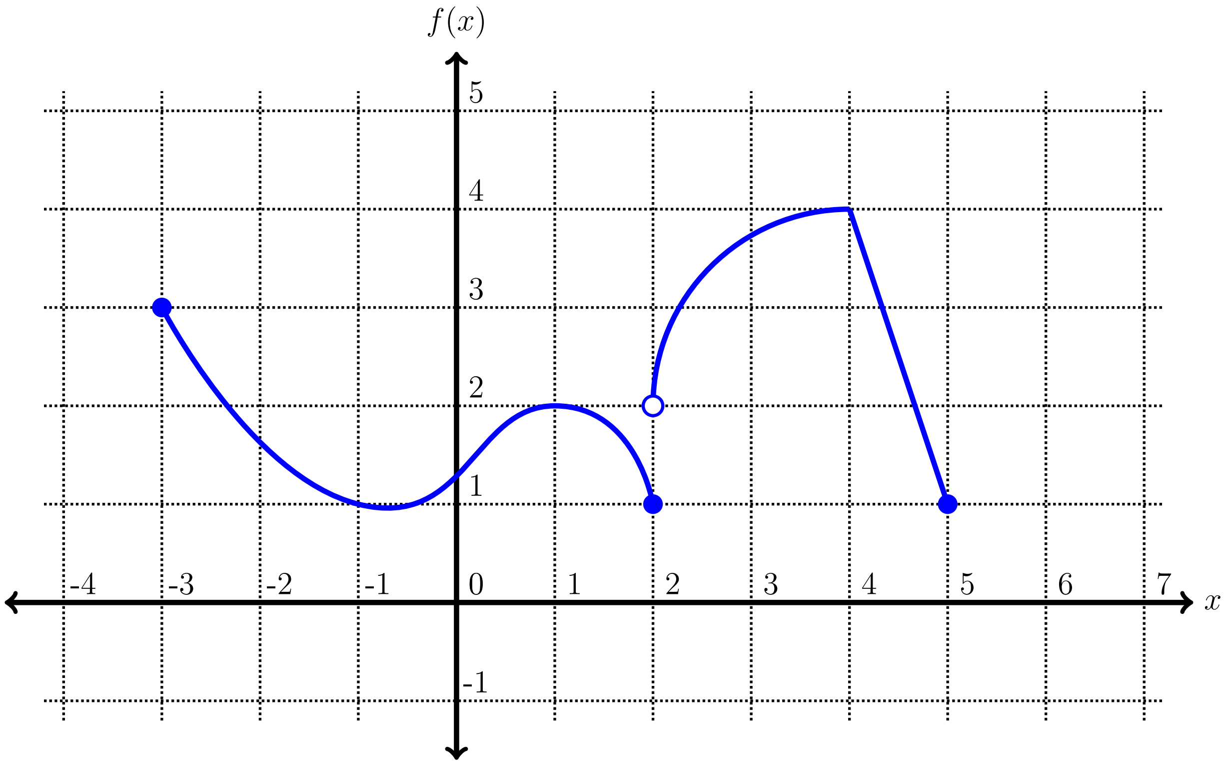
DFEP #16: Monday, February 23rd.

Here's a graph of $f(x)$. Sketch a graph of $f(-x+3)$.

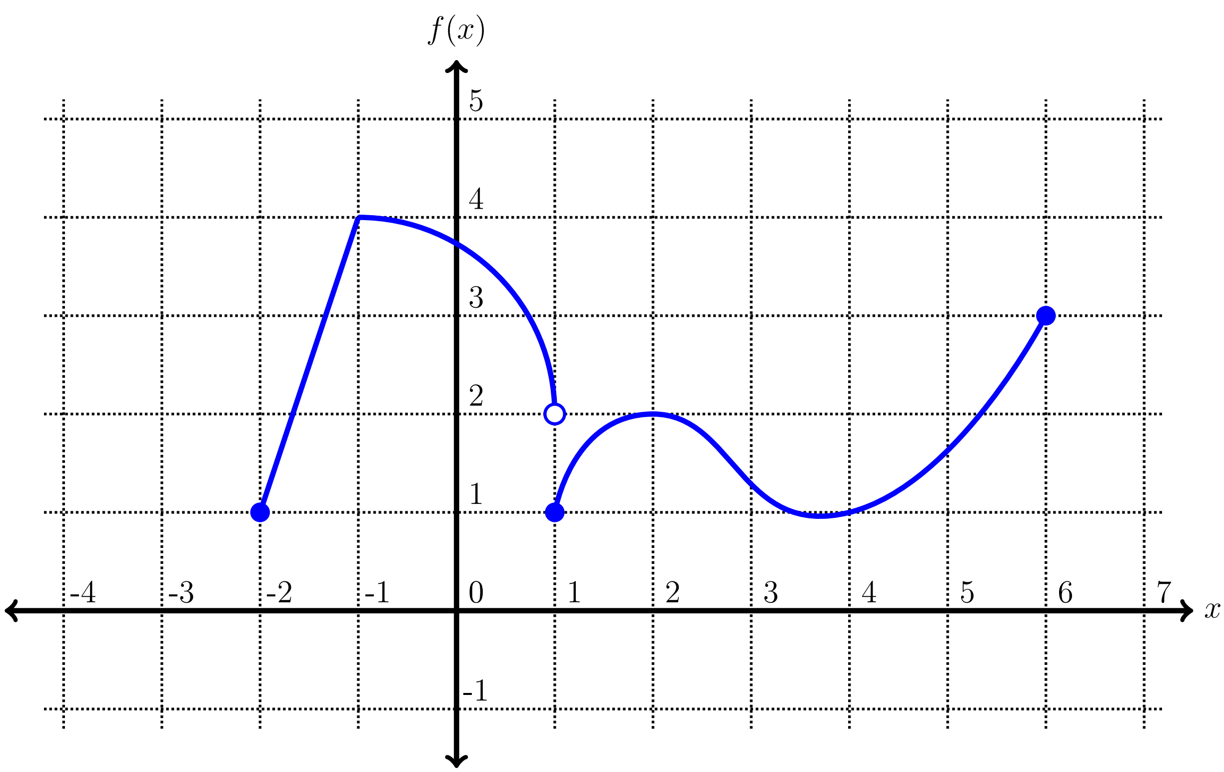


DFEP #16 Solution:

$f(-x+3)$ is the graph of $f(x)$, first moved three units to the left, and then reflected over the y-axis. So we go from this:

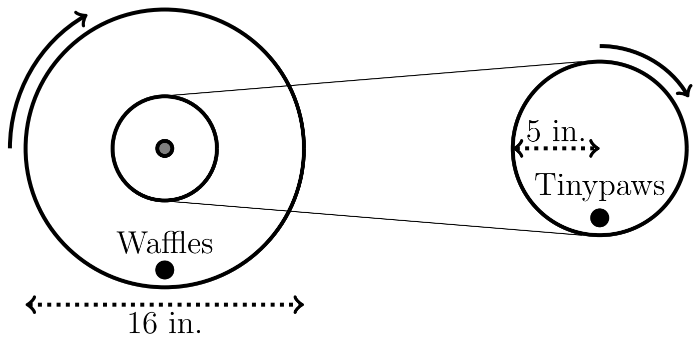


To this:

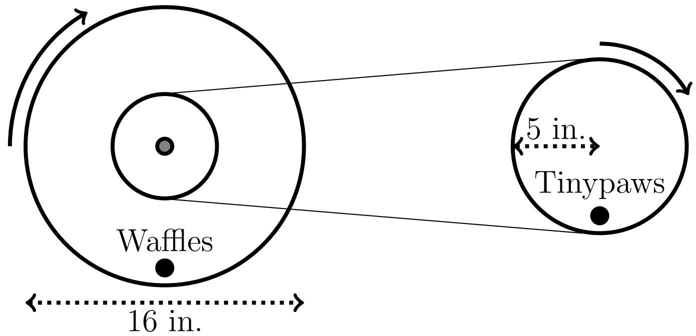


DFEP #17: Friday, February 27th.

Waffles and Admiral Tinypaws are running inside two wheels that are connected through a series of belts and axles. Waffles's wheel is 16 inches in diameter and is connected by an axle to a smaller wheel. That smaller wheel is connected by a belt to Admiral Tinypaws's wheel, which has a radius of 5 inches and makes one complete rotation every 15 seconds. If Waffles is running at a speed of 9 inches per second, what is the radius of the smallest wheel?



DFEP #17 Solution:



Let's make a table to represent the three wheels. We know that Waffles's wheel has a radius of 8 inches and rotates at a linear speed of 9 inches per second, and Admiral Tinypaws's wheel has a radius of 5 inches and has an angular speed of $2\pi/15$, so we get this:

Wheel	v	ω	r
Waffles's wheel	9		8
Inner wheel			?
Tinypaws's wheel		$2\pi/15$	5

We can fill out the rest of this table by using two facts: across each row, we know that $v = \omega r$; and down each column, values in the indicated cells should be equal. So we quickly get the remaining values:

Wheel	v	ω	r
Waffles's wheel	9	$9/8$	8
Inner wheel	$2\pi/3$	$9/8$	$16\pi/27$
Tinypaws's wheel	$2\pi/3$	$2\pi/15$	5

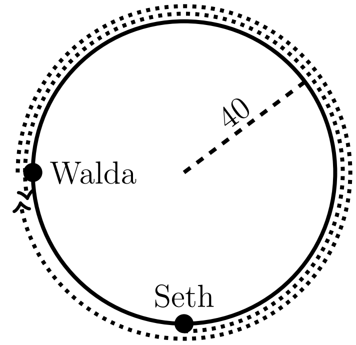
So the radius of the inner wheel is $16\pi/27 \approx 1.862$ inches.

DFEP #18: Monday, March 2nd.

Walda and Seth stand at the westernmost and southernmost points, respectively, of a circular track with radius 40 meters. At time $t = 0$, Walda begins running clockwise at a speed of 4 meters per second, while Seth walks counterclockwise at a speed of 2 meters per second. When do they pass each other for the **second** time?

DFEP #18 Solution:

Here's a picture. We don't really know where they meet the second time, but it looks something like this:



Notice that three quarters of the track is covered twice, while one quarter of the track is covered once. So, collectively, they went around the track 1.75 times. So the total distance they ran is $1.75 \cdot 2\pi 40$. If Walda went at a speed of 4 meters per second and Seth walked at a speed of 2 meters per second, then in t seconds they cover $4t + 2t = 6t$. So:

$$6t = 1.75 \cdot 2\pi 40$$

Therefore $t = \dfrac{1.75 \cdot 2\pi 40}{6} \approx 73.304$ seconds.

DFEP #19: Wednesday, March 4th.

Joaquin stands on a circular track and begins running clockwise at a speed of 13 feet per second. After 9 seconds, he reaches the northernmost point on the track. 14 seconds later, he reaches the southernmost point on the track. One minute after he starts running, how far is he (in a straight line) from his starting location?

DFEP #19 Solution:

It takes 14 seconds for Joaquin to run from the northernmost point of the track to the southernmost point of the track, so his angular speed is $\omega = \pi/14$ radians per second.

Furthermore, it takes him 9 seconds of clockwise running to move from his starting location (θ_0) to the northernmost point on the track ($\pi/2$), so $\theta_0 - 9\omega = \pi/2$, and therefore $\theta_0 = \pi/2 + 9\pi/14 = 8\pi/7$.

Finally, since we know Joaquin's linear speed is 13 feet per second and his angular speed is $\pi/14$ radians per second, we can use $v = \omega r$ to find the radius of the track to be $182/\pi$ feet.

Let's center the track at the origin. So after t seconds, his coordinates are

$$x(t) = \frac{182}{\pi}\cos\left(\frac{8\pi}{7} - \frac{\pi}{14}t\right)$$

$$y(t) = \frac{182}{\pi}\sin\left(\frac{8\pi}{7} - \frac{\pi}{14}t\right)$$

At the beginning (time $t = 0$), that comes out to $(-52.195, -25.136)$. After one minute (time $t = 60$), that's $(-52.195, 25.136)$. The distance between these two points can be found using the distance formula or, more easily, by noting that they have the same x-coordinate and just taking the differences of the y-coordinates.

The final answer is 50.272 feet.

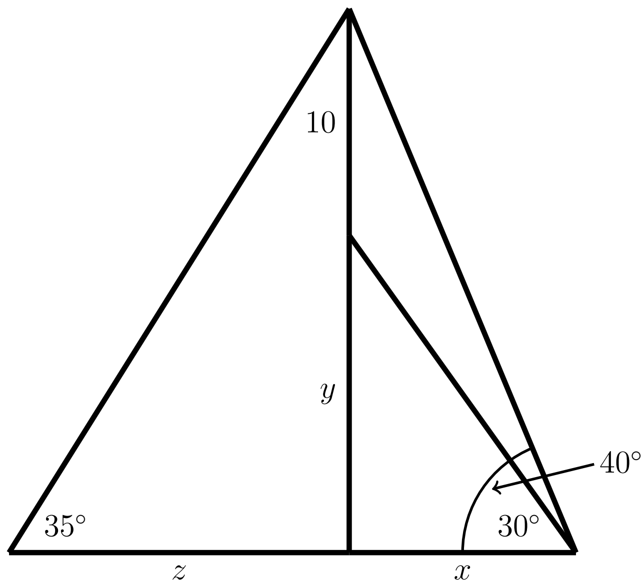
DFEP #20: Friday, March 6th.

Standing on the ground, you see a flag part way up a flagpole at an angle of elevation $30°$ from the horizontal. Then the flag is hoisted 10 feet higher into the air, and now the angle of elevation is $40°$.

If someone else sees the hoisted flag at an angle of $35°$, how far are they away from the flag pole?

DFEP #20 Solution:

Here's a picture:



We're trying to find z, and we have three equations:

$\tan(30°) = y/x \qquad \tan(40°) = (y + 10)/x \qquad \tan(35°) = (y + 10)/z.$

Take the first two equations, solve them both for x, and set them equal to each other to get

$$\frac{y}{\tan(30°)} = \frac{y + 10}{\tan(40°)}$$

Solving for y gives you $y = \dfrac{10\tan(30°)}{\tan(40°) - \tan(30°)} \approx 22.057$ feet, and plugging this into the third equation and solving for z gives

$$z = \frac{y + 10}{\tan(35°)} \approx 45.783 \text{ feet}$$

DFEP #21: Monday, March 9th.

This weird tree's height is a sinusoidal function of time. Look, I dunno, it's because of the moon or something. You try coming up with a realistic word problem every day.

Three hours ago its height was at a minimum of 10 feet tall. It'll reach its maximum height of 12 feet two hours from now. How tall was it four hours ago?

DFEP #21 Solution:

Let's let $f(t)$ be the height of the tree as a function of time.

The minimum and maximum heights are 10 and 12 feet, so, $A = \frac{12-10}{2} = 1$.

It takes 5 hours to go from the minimum to maximum height, so $B = 10$.

The maximum height is at $t = 2$, so $C = 2 - \frac{B}{4} = -0.5$.

And the average height is $D = \frac{12+10}{2} = 11$.

So, $f(t) = 1\sin\left(\frac{2\pi}{10}(x+0.5)\right) + 11$, and $f(-4) = 10.109$ feet.

DFEP #22: Wednesday, March 11th.

A weight attached to a spring moves back and forth along a frictionless surface. At $t = 3$ seconds, the spring is at its maximum length of 7 meters. The next time it reaches its minimum length of 2 meters is at time $t = 10$ seconds.

In the first 30 seconds, for how long is the spring's length greater than 3 meters?