This project is optional. Your score on this project will be 100%; the quality will determine how much of your grade it accounts for, in the range of 0% to 5%. The first project will account for the remainder of the 10% project portion.

In this project, you'll think about the possible shapes of some rational functions.

Let $f(x)$ be a rational function of the form $\dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials of degree ≤ 2.

In other words, for some real numbers a_2, a_1, a_0, b_2, b_1, b_0 (some of which can be zero):

$$f(x) = \frac{a_2 x^2 + a_1 x + a_0}{b_2 x^2 + b_1 x + b_0}$$

For each of the following prompts, I want you to tell me whether it's possible for $f(x)$ to have the given properties. Remember that a_2, a_1, a_0, b_2, b_1, and b_0 can be any real numbers.

If your answer is "no", you should explain (in words, equations, and/or pictures) why that's impossible.

If your answer is "yes", you should give an example of such a function, *and* explain why that answer works. "Because I plugged it into a graphing calculator" isn't a good answer; you should show your work like you would on an exam.

    Is it possible...

(a) ...that $f(x)$ has three x-intercepts?

(b) ...that $f(x)$ has no horizontal asymptote?

(c) ...that $f(x)$ has two different horizontal asymptotes?

(d) ...that $f(x)$ has one vertical asymptote and one removable discontinuity?

(e) ...that $f(x)$ is tangent to the line $y = 2x + 1$?

(f) ...that $f(x)$ has an absolute minimum *and* an absolute maximum over the interval $(-\infty, \infty)$?

(g) ...that $f(x)$ has three local extrema?

(h) ...that $f(x)$ has two local maxima?

(i) ...that $f(x)$ has three points of inflection?

(j) ...that $f(x)$ is tangent to a single line in exactly **two** places?

Rules and Regulations

(1) You should work on your own. Please don't make me suspicious of copying.

(2) Your writeup may be handwritten or typed, or a combination of both, but it should be **easy to read**. You probably want to work through the problems first on a separate piece of paper, and then write up your cleanest solutions.

(3) Show all your work, but you don't have to be purposely longwinded. Brevity is good in mathematics. Overall, your goal is to be **clear**.

(4) Use sentences when you want to explain something. Use calculations when you want to compute something. Use pictures when you want the reader to visualize something. The best mathematics papers include a mix of all three.