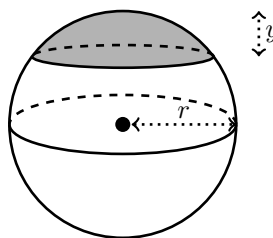


In this project, you'll answer a bunch of questions about how far you can see while standing on a planet. Just to be clear, here's the deal: you're standing on a perfect sphere of radius r . Your height—or rather, the distance from your eyes to the ground at your feet—is h . You can see in a straight line, and you can't see through the planet because it's opaque.

1. So, how far can you see? (Your answers should involve both h and r .) Give three different answers:
 - (a) How far can you see to the horizon **in a straight line**? That is, find the straight-line distance from your eyes to the farthest point on the planet you can see.
 - (b) How far can you see to the horizon **along the planet**? This time you want the length of an arc along the planet from your feet to the farthest point you can see.
 - (c) What **area** can you see? Give the total surface area of the planet that's visible to you. Maybe you don't know the formula for the surface area of a spherical cap, so here's a picture. The area of the gray yarmulke-shaped region is $2\pi r y$:



2. Compute specific values for each of the three quantities in #1, where h is your height and r is the radius of the Earth. (Watch out for units!)
3. For each of the three quantities in #1, take both the limit as h goes to zero, and the limit as h goes to infinity. Let r be constant.
For all six limits, you should show your calculations and briefly explain why your answer makes sense, intuitively.
4. For each of the three quantities in #1, compute the derivative with respect to r , and the derivative with respect to h .
For all six derivatives, show your calculations and briefly explain what they represent.
5. **Ask your own question here, based on the problems above.** If you can, answer it! If you can't, that's okay, but discuss how one might solve it, or what difficulties would be involved.

Rules and Regulations

- (i) You should work on your own. Please don't make me suspicious of copying.
- (ii) Your writeup may be handwritten or typed, or a combination of both, but it should be **easy to read**. You probably want to work through the problems first on a separate piece of paper, and then write up your cleanest solutions.
- (iii) Show all your work, but you don't have to be purposely longwinded. Brevity is good in mathematics. Overall, your goal is to be **clear**.
- (iv) Use sentences when you want to explain something. Use calculations when you want to compute something. Use pictures when you want the reader to visualize something. The best mathematics papers include a mix of all three.
- (v) I expect your work will take up between 2 and 4 pages, but it depends how large your writing is. There's no official minimum or maximum.