HW #2: Chapter 2.1 (6367092)

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0/120</td>
<td>10</td>
<td>14</td>
<td>14</td>
<td>0/4</td>
<td>0/45</td>
</tr>
</tbody>
</table>

1. 0/12 points

A tank holds 5000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

<table>
<thead>
<tr>
<th>t (min)</th>
<th>V (gal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3425</td>
</tr>
<tr>
<td>10</td>
<td>2185</td>
</tr>
<tr>
<td>15</td>
<td>1300</td>
</tr>
<tr>
<td>20</td>
<td>505</td>
</tr>
<tr>
<td>25</td>
<td>145</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) If P is the point (15, 1300) on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with the following values. (Round your answers to one decimal place.)

<table>
<thead>
<tr>
<th>Q</th>
<th>slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5, 3425)</td>
<td></td>
</tr>
<tr>
<td>(10, 2185)</td>
<td></td>
</tr>
<tr>
<td>(20, 505)</td>
<td></td>
</tr>
<tr>
<td>(25, 145)</td>
<td></td>
</tr>
<tr>
<td>(30, 0)</td>
<td></td>
</tr>
</tbody>
</table>

(b) Estimate the slope of the tangent line at P by averaging the slopes of two adjacent secant lines. (Round your answer to one decimal place.)

2. 0/11 points

EXAMPLE 1  Find an equation of the tangent line to the function y = 5x^2 at the point P(1, 5).

SOLUTION  We will be able to find an equation of the tangent line t as soon as we know its slope m. The difficulty is that we know only one point, P, on t, whereas we need two points to compute the slope. But observe that we can compute an approximation to m by choosing a nearby point Q(x, 5x^2) on the graph (as in the figure) and computing the slope m_{PQ} of the secant line PQ. [A secant line, from the Latin word secans, meaning cutting, is a line that cuts (intersects) a curve more than once.]

We choose x ≠ 1 so that Q ≠ P. Then,

\[ m_{PQ} = \frac{5x^2 - 5}{x - 1} \]

For instance, for the point Q(1.5, 11.25) we have

\[ m_{PQ} = \frac{11.25 - 5}{0.5} = \frac{6.25}{0.5} = 12.5. \]

The tables below show the values of m_{PQ} for several values of x close to 1. The closer Q is to P, the closer x is to 1 and, it appears from the tables, the closer m_{PQ} is to . This suggests that the slope of the tangent line t should be m = .

<table>
<thead>
<tr>
<th>x</th>
<th>m_{PQ}</th>
<th>x</th>
<th>m_{PQ}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
We say that the slope of the tangent line is the limit of the slopes of the secant lines, and we express this symbolically by writing

$$\lim_{Q \to P} m_{PQ} = m$$

and

$$\lim_{x \to 1} \frac{5x^2 - 5}{x - 1} = \text{[value]}$$

Assuming that this is indeed the slope of the tangent line, we use the point-slope form of the equation of a line (see Appendix B) to write the equation of the tangent line through (1, 5) as

$$y - 5 = m(x - 1) \quad \text{or} \quad y = \left(\frac{5}{x - 1}\right) x - 5$$

The graphs below illustrate the limiting process that occurs in this example. As Q approaches P along the graph, the corresponding secant lines rotate about P and approach the tangent line t.

3. 0/14 points fallingladderhw1 [1225427] .

A ladder 25 feet long is leaning against the wall of a building. Initially, the foot of the ladder is 7 feet from the wall. The foot of the ladder begins to slide at a rate of 2 ft/sec, causing the top of the ladder to slide down the wall. The location of the foot of the ladder at time t seconds is given by the parametric equations (7+2t,0).

(a) The location of the top of the ladder will be given by parametric equations (0,y(t)). The formula for y(t)=

. (Put your cursor in the box, click and a palette will come up to help you enter your symbolic answer.)
(b) The domain of $t$ values for $y(t)$ ranges from __________ to __________

(c) Calculate the average velocity of the top of the ladder on each of these time intervals (correct to three decimal places):

<table>
<thead>
<tr>
<th>time interval</th>
<th>ave velocity</th>
<th>time interval</th>
<th>ave velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0,2]</td>
<td>__________</td>
<td>[2,4]</td>
<td>__________</td>
</tr>
<tr>
<td>[6,8]</td>
<td>__________</td>
<td>[8,9]</td>
<td>__________</td>
</tr>
</tbody>
</table>

(d) Find a time interval $[a,9]$ so that the average velocity of the top of the ladder on this time interval is -20 ft/sec i.e. $a=________$

(e) Using your work above and this picture of the graph of the function $y(t)$ given below, answer these true/false questions: (Type in the word "True" or "False")

The top of the ladder is moving down the wall at a constant rate

- T
- F

The foot of the ladder is moving along the ground at a constant rate

- T
- F

There is a time at which the average velocity of the top of the ladder on the time interval $[a,9]$ is 1 ft/sec

- T
- F

There is a time at which the average velocity of the top of the ladder on the time interval $[a,9]$ is 0 ft/sec

- T
- F

There is a time at which the average velocity of the top of the ladder on the time interval $[a,9]$ is -100 ft/sec

- T
- F

There is a time at which the average velocity of the top of the ladder on the time interval $[a,9]$ is less than -100 ft/sec

- T
- F
4. 0/4 points

The cup on the 9th hole of a golf course is located dead center in the middle of a circular green which is 30 feet in radius. Your ball is located as in the picture below. The ball follows a straight line path and exits the green at the right-most edge. Assume the ball travels 11 ft/sec. Introduce coordinates so that the cup is the origin of an \(xy\)-coordinate system. Provide numerical answers below with two decimal places of accuracy.

(a) The \(x\)-coordinate of the position where the ball enters the green will be [__________].

(b) The ball will exit the green exactly [__________] seconds after it is hit.

(c) Suppose that \(L\) is a line tangent to the boundary of the golf green and parallel to the path of the ball. Let \(Q\) be the point where the line is tangent to the circle. Notice that there are two possible positions for \(Q\). Find the possible \(x\)-coordinates of \(Q\):

   smallest \(x\)-coordinate = [__________]
   largest \(x\)-coordinate = [__________]

5. 0/4 points

A Ferris wheel of radius 100 feet is rotating at a constant angular speed \(\omega\) rad/sec counterclockwise. Using a stopwatch, the rider finds it takes 6 seconds to go from the lowest point on the ride to a point \(Q\), which is level with the top of a 44 ft pole. Assume the lowest point of the ride is 3 feet above ground level.

Let \(Q(t) = (x(t), y(t))\) be the coordinates of the rider at time \(t\) seconds; i.e., the parametric equations. Assuming the rider begins at the lowest point on the wheel, then the parametric equations will have the form: \(x(t) = r\cos(\omega t - \pi/2)\) and \(y(t) = r\sin(\omega t - \pi/2)\), where \(r, \omega\) can be determined from the information given. Provide answers below accurate to 3 decimal places. (Note: We have imposed a coordinate system so that the center of the ferris wheel is the origin. There are other ways to impose coordinates, leading to different parametric equations.)

(a) \(r = [\text{__________}]\) feet

(b) \(\omega = [\text{__________}]\) rad/sec

(c) During the first revolution of the wheel, find the times when the rider's height above the ground is 80 feet.

   first time = [\text{__________}] sec
   second time = [\text{__________}] sec