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**A List of Topics for the Final**

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Here's what you should be able to do for the final next week.

**Really old stuff.**

## 1. Limit rules

- (a) Calculate limits using basic properties: if you know the limits of two expressions, can you find the limits of their sum, product, etc.?
- (b) Compute more difficult limits using cancellation, multiplication by the conjugate, and other algebraic tricks.
- (c) Recognize when limits tend to infinity or do not exist.
- (d) Do all of the above when  $x$  tends to  $\infty$  or  $-\infty$  rather than some real number  $a$ .

## 2. Basic derivatives

- (a) Understand the relationship between derivatives and limits, and compute basic derivatives by evaluating limits.
- (b) Compute limits of monomials (via the power rule),  $e^x$ , and trigonometric functions.
- (c) Use the product rule and quotient rule to find derivatives of functions that are products and quotients of other functions.
- (d) Find the equation for a tangent line to a function at a certain point.

**Old stuff.**

## 3. Advanced derivatives

- (a) Compute the derivatives of exponential, inverse trigonometric, and logarithmic functions.
- (b) Use the chain rule to find the derivatives of compositions of functions.
- (c) Use logarithmic differentiation to differentiate functions of the form  $f(x) = g(x)^{h(x)}$ .

## 4. Calculus with parametric equations

- (a) Find the equation for a tangent line to a parametric curve at a given point in time, or at a given point on the curve.
- (b) Determine when a particle is moving horizontally, or vertically, or not moving at all, by examining the derivatives of its parametric equations.
- (c) Calculate the speed of an object based on its parametric equations.

## 5. Implicit differentiation

- (a) Compute  $y'$ ,  $y''$ ,  $y'''$ , etc. when  $x$  and  $y$  are related by an implicit equation.
- (b) Find the equation for the tangent line to a given point on a curve.
- (c) Find all points on a curve whose tangent line is horizontal or vertical.

- (d) Find the equation for a tangent line to a given curve that passes through a given point **not** on that curve.

6. Related rates

- (a) Solve related rates problems. Sorry if you were expecting something really deep.

7. Linear approximation

- (a) Find the linear approximation to a function around a certain point.
- (b) Use linear approximation to estimate values of functions that would otherwise be difficult without the aid of technology.
- (c) Use linear approximation and implicit differentiation to estimate solutions to an implicit equation that would otherwise be difficult or impossible to solve algebraically.

**New-ish Stuff**

8. The shape of a function

- (a) Calculate the absolute minimum and maximum values of a function on a closed interval.
- (b) Find the critical points of a function, determine where it's increasing, decreasing, concave up, and concave down.
- (c) Identify local extrema, points of inflection, asymptotes, and intercepts.
- (d) Graph all that stuff.
- (e) Answer conceptual problems about the relationship between  $f$ ,  $f'$ , and  $f''$  involving the above concepts.

9. l'Hôpital's rule

- (a) Use l'Hôpital's rule to compute limits of the form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ .
- (b) Convert limits in other indeterminate forms ( $0 \cdot \infty$ ,  $\infty - \infty$ ,  $1^\infty$ ,  $\infty^0$ , and  $0^0$ ) into the form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$  (in various ways), so that you can use l'Hôpital's rule.
- (c) Spell and pronounce "l'Hôpital".

10. Optimization

- (a) Solve optimization problems.
- (b) Yeah okay, more specifically: you should be able to find the absolute minimum and maximum of a function *even if* the interval isn't closed, by using the first or second derivative tests or common sense.