A List of Topics for the Second Midterm

Here's a fairly comprehensive list of things you should be comfortable doing for the second midterm.

Old Stuff

1. Unit conversion and rates of change.
2. Coordinate systems.
   (a) Plotting things in a coordinate system.
   (b) Using the distance formula.
3. Equations for lines and circles.
   (a) Finding intersections of curves.
   (b) Writing equations for circles and semicircles.
4. Linear modeling.
   (a) Finding an equation for a line given various pieces of information. Finding the shortest distance from a line to a point not on that line.
   (b) Using linear equations for real-world problems with constant rates of change.
   (c) Finding parametric equations for linear motion.
5. Functions and graphing.
   (a) Graphing a function, and analyzing a function based on its graph.
   (b) Evaluating functions, and solving equations like $f(2x+3) = x$.
6. Graphical analysis.
   (a) Determining the domain and range of a function, visually or algebraically, and using the vertical line test.
   (b) Graphing, constructing, and solving multipart functions.

New Stuff

7. Quadratic modeling.
   (a) Graphing quadratic functions and converting to vertex form.
   (b) Finding the minimum and/or maximum values of quadratic functions over certain ranges.
   (c) Finding a formula for a quadratic function through a given set of points, and/or with a given vertex or line of symmetry.
   (d) Constructing a quadratic to find the minimum and maximum values of certain real-world functions.

8. Functional composition.
   (a) Giving a formula for $f(g(x))$ based on the formulas for $f(x)$ and $g(x)$.
   (b) Determining the domain and range of the composition of functions.
   (c) Computing $f(g(x))$ when f and/or g are multipart functions.
   (d) Computing "fixed points" of a function $f(x)$. That is, finding solutions to the equation $f(x) = x$.
9. Inverse functions.
   (a) Computing the inverse of a function algebraically, and drawing the inverse of a function graphically.
   (b) Determining whether a function is one-to-one, both algebraically and graphically.
   (c) For certain functions that *aren't* one-to-one (e.g. parabolas), knowing how to break those functions down into smaller parts, and finding inverses for each of those pieces.
10. Exponential functions.
    (a) Computing and manipulating exponential functions.
    (b) Knowing the various rules of exponents.
    (c) Converting exponential functions into "standard exponential form".
11. Exponential modeling.
    (a) Finding an exponential function to match real-world data.
    (b) Computing discretely or continuously compounded interest rates.
12. Logarithmic functions.
    (a) Relating logarithms to exponential functions, and using them to solve exponential equations.
    (b) Manipulating said equations by the properties of logarithmic functions.
    (c) Graphing logarithmic functions.
13. Graphical transformations.
    (a) Manipulating an equation algebraically in order to translate, reflect, and/or dilate its graph.
    (b) Drawing a graph based on an equation, after it has had the above transformations applied.
14. Rational functions.
    (a) Graphing linear-to-linear rational functions and computing their asymptotes.
    (b) Finding a linear-to-linear rational function based on data points and/or asymptotes.
    (c) Using linear-to-linear rational functions to model real-world problems.

Some Useful Equations

- The distance d between points (x_1, y_1) and (x_2, y_2): $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- A line through points (x_1, y_1) and (x_2, y_2): $y = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1$
- A line through the point (x_1, y_1) with slope m: $y = m(x - x_1) + y_1$
- A line with y-intercept b and slope m: $y = mx + b$
- A circle with center (x_0, y_0) and radius r: $(x - x_0)^2 + (y - y_0)^2 = r^2$
- The parametric equations for uniform linear motion from (x_0, y_0) to (x_1, y_1) in Δt units of time, where $\Delta x = x_1 - x_0$, and $\Delta y = y_1 - y_0$:

$$x = x_0 + \frac{\Delta x}{\Delta t} t \qquad y = y_0 + \frac{\Delta y}{\Delta t} t$$

- An upper semicircle with center (x_0, y_0) and radius r: $y = y_0 + \sqrt{r^2 - (x - x_0)^2}$
- A lower semicircle with center (x_0, y_0) and radius r: $y = y_0 - \sqrt{r^2 - (x - x_0)^2}$
- A quadratic, with vertex (h, k) and scaling factor a: $y = a(x - h)^2 + k$
- Converting to vertex form from $y = ax^2 + bx + c$: $h = \dfrac{-b}{2a}$ $\quad k = c - \dfrac{b^2}{4a}$
- An exponential with starting value A_0 and annual growth factor b: $y = A_0 b^x$
- Properties of exponential functions and logarithms:

$$b^x b^y = b^{x+y} \qquad \frac{b^x}{b^y} = b^{x-y} \qquad \left(b^x\right)^y = b^{xy}$$

$$(ab)^x = a^x b^x \qquad b^{-x} = \frac{1}{b^x} \qquad b^0 = 1$$

$$\ln(xy) = \ln(x) + \ln(y) \qquad \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y) \qquad \ln(x^y) = y\ln(x)$$

$$\log_b(x) = \frac{\ln(x)}{\ln(b)} \qquad \ln(e^x) = x \qquad e^{\ln(x)} = x$$

- A linear-to-linear rational function, asymptotes $y = a$ and $x = -d$: $y = \dfrac{ax + b}{x + d}$