Reading assignment: Review §5.3; read the lecture notes as well as §§8.1 and 8.2 in the book; then skim through §8.5.

Written assignments:

1. There are 100 towns in a country and some of them are connected by airlines. It is known that one can reach every town from any other (perhaps with several intermediate stops). Prove that you can fly around the country and visit ALL the towns making no more than 198 flights.
   Hint: Consider a spanning tree of the graph of routes. Double each edge of this tree; then apply Euler's theorem.

2. Let G be a **connected** graph with at least two vertices. Prove that G has a vertex v such that if v is removed from G (along with all edges incident with it), the resulting graph is also connected.
   Hint: Consider a spanning tree and one of its leaves.

3. (a) Find the number of trees on the vertex set $V = \{1, 2, \dots, 8\}$ with the property that $\deg(1) = \cdots = \deg(6) = 1$ and $\deg(7) = \deg(8) = 4$.
   (b) Find the number of trees on the vertex set $V = \{1, 2, \dots, 8\}$ in which all vertices have degree 1 or 4.

4. Determine how many among the spanning trees of K_n (a complete graph on n vertices) have vertex n as a leaf.
   Hint: Cayley's formula.

5. Show that the number of (non-equal) forests on the vertex set $V = \{1, 2, \dots, n\}$ that contain exactly two connected components is given by the formula $\sum_{k=1}^{n-1} \binom{n-1}{k-1} k^{k-2}(n-k)^{n-k-2}$.
   Hint: consider the connected component of 1.

6. Prove that the number of (non-equal) trees on the vertex set $V = \{1, 2, \dots, n\}$ that contain the edge $\{1, 2\}$ is $2n^{n-3}$.
   Hint: notice that every pair $\{i, j\} \subset V$ appears as an edge in exactly the same number of trees. Use this observation to determine which fraction of all the trees contains a given pair as an edge.