Reading assignment: Read lecture notes (Lectures 17–19), review §4.2, read §4.3 (you may skip Theorem 4.3.3), and start looking at §4.4.

Written assignments:

1. Do the first six parts of #2 from §4.1. Please explain your answers: if you claim that a statement is false — provide a counter-example, and if you claim that the statement is true — provide a short proof. Note that there is a typo in part (a): the very end of part (a) should read: $\{u, v\} \in E(G) \Leftrightarrow \{f(u), f(v)\} \in E(H)$.

2. Is it true that two graphs must be isomorphic if

   (a) They both have 10 vertices and the degree of each equals 9?

   (b) They both have 8 vertices and the degree of each equals 3?

   (c) They are both connected, without cycles, and have 6 edges?

   Please explain your answers.

3. Ten players participate at a chess tournament. Eleven games have already been played. Prove that there is a player who has played at least 3 games.

4. In a certain country, 100 roads lead out of each city, and one can travel along those roads from any city to any other. One road is closed for repairs. Prove that one can still get from any city to any other.
   Hint: Assume the road between cities A and B was closed. If after this closure there is no way to reach B from A, how many odd-degree vertices are in the connected component of A?

5. Is there a disconnected graph with degree sequence (4, 4, 3, 3, 3, 3, 3, 3)?
   Hint: what is the minimal possible number of vertices in the connected component of a vertex of degree d?

6. Let G be a graph. The **complement of** G, $\overline{G}$, is the graph with the same vertex set as G whose edges "complement" the edges of G, that is, $\{i, j\}$ is an edge of $\overline{G}$ if and only if $\{i, j\}$ is NOT an edge of G.

   Show that if G is a disconnected graph, then $\overline{G}$ is connected.