Reading assignment:

• *After Friday's lecture (Oct 7)*, read Lecture 5 notes as well as as §1.2 and §1.4 in the "Invitation to Discrete Math" book. (The material in these two sections must be familiar from previous math classes, but it might be helpful to review some definitions and notation. We'll need some of these notions for our next topic — "Combinatorial counting".) Then start reading §3.1.

• *After Monday's lecture (Oct 10)*, read Lecture 6 notes, finish reading §3.1 and start reading §3.2 in the "Invitation to Discrete Math" book.

• *After Wednesday's and Friday's lectures (Oct 12 and 14)*, read Lecture 7 and Lecture 8 notes, finish reading §3.2, and start reading §3.3 (but skip page 69 for now, except for the very bottom of this page).

Written assignment: Solve the following problems. Please don't forget to follow the Homework Guidelines (see our web-page). If you use the Pigeonhole Principle, tell us what your pigeons and what your pigeonholes are, and how do you assign a pigeon to a pigeonhole.

1. Five points with integer coordinates are chosen on a plane. Prove that the midpoint of one of the segments joining two of these points also has integer coordinates.

2. We are given 17 points inside an equilateral triangle of side length one. Prove that there are two points among them whose distance is not more than 1/4.

3. Prove that out of any 64 whole numbers, one can always choose ten so that the difference of every two chosen numbers is divisible by seven.

4. How many four-digit positive integers are there in which all digits are different? (We do not allow the first digit to be zero.)

5. How many four-digit numbers are there in which the sum of the digits is even? (We do not allow the first digit to be zero.)

6. A class is attended by n sophomores, n juniors, and n seniors. In how many ways can these students form n groups of three people each if each group is to contain a sophomore, a junior, and a senior?

7. We want to select as many subsets of $[n] := \{1, 2, \ldots, n\}$ as possible so that any two selected subsets have at least one element in common (e.g., the collection consisting of $\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$ satisfies this property). What is the largest number of subsets we can select?
   Note: if your answer is $X(n)$, you'll need to prove two statements: (i) that $X(n)$ such subsets exist, and that (ii) no collection of $X(n) + 1$ or more subsets has the required property. For this latter statement, the Pigeonhole Principle might be useful.

8. Do #2 from §3.1 (feel free to read the hint on p. 409).