Homework assignments in this class will be usually due on Friday. *The importance of doing all the homework cannot be overemphasized.* Mathematics cannot be learned passively. Even professional mathematicians frequently find that something can look straightforward, even easy, when someone else is demonstrating it, but turn out to be confusing and difficult when first attempted, and require a fair amount of practice to master.

Reading assignment:

• *After Wednesday's lecture (Sept 28)*, read Lecture 1 notes (to be posted later today) as well as Section 1.1 from "Invitation to Discrete Mathematics". Glance through Section 1.3.

• *After Friday's lecture (Sept 30)*, read Lecture 2 notes as well as Section 1.3 in the book.

• *After Monday's lecture (Oct 3)*, read Lecture 3 notes. You may also want to take a look at §2.4 in the book by L. Lovász, J. Pelikán, and K. Vesztergombi.

• *After Wednesday's lecture (Oct 5)*, read Lecture 4 notes as well as §1.2 and §1.4 in the "Invitation to Discrete Math" book. (The material in these two sections must be familiar from previous math classes, but it might be helpful to review some definitions and notation. We'll need some of these notions for our next topic — "Combinatorial counting".)

Written assignment: Solve the following problems. Please don't forget to follow the Homework Guidelines (see our web-page). If you define a graph, tell us explicitly what are the vertices and edges of your graph. If you use the Pigeonhole Principle, tell us what your pigeons and what your pigeonholes are, and how do you assign a pigeon to a pigeonhole. Explain your computations.

1. (A warm-up problem) If a graph has n vertices, what is the smallest possible degree of any vertex? What is the largest possible degree of any vertex? Explain!
2. How many diagonals does an n-gon have?
3. (a) In Smallville there are 15 telephones. Can they be connected by wires so that each telephone is directly connected with exactly five others?
   (b) Can a kingdom in which 13 roads lead out of each city have exactly 10000 roads?
4. (This is problem #1 from §1.3) Prove the following formulas by mathematical induction.
   (a) $1 + 2 + \cdots + n = n(n+1)/2$
   (b) $\sum_{i=1}^{n} i \cdot 2^i = (n-1)2^{n+1} + 2$.
5. Let $a_0 = 0$, $a_1 = 1$, and let $a_{n+2} = 6a_{n+1} - 9a_n$ for $n \geq 0$. Prove that $a_n = n \cdot 3^{n-1}$ for all $n \geq 0$.
6. Prove that for any positive integer n, it is possible to partition any triangle T into $3n + 1$ similar triangles.
7. Do #11 from §1.3.
8. Each box in a 3×3 arrangement of boxes is filled with one of the numbers $-1, 0, 1$. Prove that of the eight possible sums along the rows, the columns, and the diagonals, two sums must be equal.