The final will take place on Wednesday, December 14, 2:30-4:20 in our usual room. The final will be comprehensive. You may bring one notebook sized sheet (up to 8.5 by 11 in.) of ***handwritten*** notes.

We'll have a review session on Tuesday, December 13, 10:30-12:30 in Padelford C-36.

To prepare for the test, make sure you know how to solve all the homework problems as well as problems from both midterms. Here are some additional problems for you to practice: the first five of them are on planar graphs — the material we have been covering this last week, the last five are review problems.

1. If G is a connected planar graph on 24 vertices and each vertex of G has degree 3, how many faces are there in a planar drawing of G?

2. (a) Let G be a planar graph on $V \geq 2$ vertices. Prove that G has at least two vertices whose degrees are at most 5.
   (b) Prove that every planar graph with fewer than 12 vertices has a vertex of degree ≤ 4.

3. Each edge of the complete graph with 11 vertices is colored either red or blue. We then look at the graph consisting of all the red edges, and the graph consisting of all the blue edges. Prove that at least one of these two graphs is not planar.

4. (a) Is it true that if a connected graph satisfies $E \leq 3V - 6$, then that graph is planar?
   (b) Take K_6, the complete graph on 6 vertices, and delete two of its edges. Prove that the obtained graph is never planar. What about 3 edges?

5. A planar graph G has 16 vertices and 40 edges. How many triangular regions are there in the planar drawing of G, if all the regions (including the unbounded one) are either triangles or quadrilaterals?

6. (Short questions)
   (a) How many strings can be written using exactly five letters A, three letters B, and no more than two letters C (and no other letters)?
   (b) We must choose a 5-member team from 12 girls and 10 boys. How many ways are there to make such a choice so that there are no more than 3 boys on the team?
   (c) How many of graphs on the vertex set $\{1, 2, \ldots, 12\}$ have exactly fifteen edges?
   (d) Let G be a connected planar graph with 100 vertices and 140 edges. How many faces are in a planar drawing of G?
   (e) Let F be a forest with 100 vertices and 90 edges. How many new edges must be added without adding vertices to obtain a tree?
   (f) Suppose that a graph G has 1000 vertices and 3000 edges. Can G be planar?
   (g) Does there exist a graph on 7 vertices whose degree sequence is $(5, 5, 4, 4, 3, 2, 2)$?
   (h) For which values of n is K_n planar? (Recall that K_n is the complete graph on n vertices.)

7. (a) Let G be a connected graph with n vertices and n edges. How many cycles does G have? Explain!

   (b) Which trees are Eulerian graphs? Explain!

8. In a room there are 10 people, none of whom are older than 100 (ages are given in whole numbers only) but each of whom is at least 1 year old. Prove that one can always find two groups of people (possibly intersecting, but different) the sums of whose ages are the same.

9. Recall that the Fibonacci sequence $F_0, F_1, F_2, \ldots$ is defined by $F_0 = F_1 = 1$ and $F_{i+1} = F_i + F_{i-1}$ for all $i \geq 1$. Prove that F_n and F_{n+1} are relatively prime for all n.

10. Determine the number of integral solutions of the equation

$$x_1 + x_2 + x_3 + x_4 = 20$$

which satisfy

$$1 \leq x_1 \leq 6,\ 0 \leq x_2 \leq 5,\ 4 \leq x_3 \leq 9,\ 2 \leq x_4 \leq 7.$$