The final will take place on Monday, March 12, 8:30-10:20. It will be a comprehensive exam. We will devote the last two classes of the quarter to reviewing the material. In addition to my regular office hours on Monday, March 5, 2-3pm in PDL C-401 and Tuesday, March 6, 5-6:30pm, I'll also have office hours on Friday, March 9, 1:30-3:30, in PDL C-416.

You are allowed to bring one 2-sided, handwritten page of notes.

Make sure you know how to solve all the problems we've discussed this quarter.

Please do NOT forget to explain/justify all of your solutions.

Here are some additional problem for you to practice.

1. How many ways are there to arrange the letters in the word "ASUNDER" so that the vowels will be in alphabetical order, as well as the consonants?
   Example: DANERUS (A-E-U, D-N-R-S)

2. (a) How many different $n \times m$ zero-one matrices (that is, n rows, m columns, and each entry is either 0 or 1) are there?
   (b) How many among those matrices do NOT have two identical rows?

3. How many solutions in nonnegative integers does the inequality $x_1 + x_2 + \cdots + x_{10} \leq 2018$ have?

4. Prove that $S(k, 2) = 2^{k-1} - 1$.
   Hint: if the two boxes were distinct, how many ways would be there to fill in the first box?

5. Show that every set of 24 integers $\{a_1, a_2, \dots, a_{24}\}$ contains a nonempty subset with the property that the sum of its elements is divisible by 24.
   Hint: consider the following sums: $a_1,\ a_1 + a_2,\ a_1 + a_2 + a_3,\ \dots,\ a_1 + a_2 + \cdots + a_{24}$.

6. In a room there are 10 people, none of whom are older than 100 (ages are given in whole numbers only) but each of whom is at least 1 year old. Prove that one can always find two groups of people (possibly intersecting, but different) the sums of whose ages are the same.
   Hint: Use the Pigeonhole Principle.

7. The sequence of Fibonacci numbers is defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$. Prove that for all $n \geq 0$, the remainders of the Fibonacci numbers F_n and F_{n+8} when divided by 3 are equal. (The first ten Fibonacci numbers are $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, $F_7 = 13$, $F_8 = 21$, $F_9 = 34$.)

8. There are 30 students in a class. Can it happen that 9 of them have 3 friends each (in the class), 11 have 4 friends each, and 10 have 5 friends each?

9. Assume that in an n-vertex tree there is one vertex of degree 2, one vertex of degree 3, one of degree 4, one of degree 5, and the remaining $n - 4$ vertices are leaves. Find n.

10. In a certain country, 100 roads lead out of each city, and one can travel along those roads from any city to any other. One road is closed for repairs. Prove that one can still get from any city to any other.
    Hint: Assume the road between cities A and B was closed. If after this closure there is no way to reach B from A, how many odd-degree vertices are in the connected component of A?

11. How many positive integers are there that are not larger than 1000 and are neither perfect squares nor perfect cubes?

12. How many permutations of $\{1, 2, \dots, 6\}$ have at least one odd number in its natural position?

13. How many strings of length 12 can we compose using letters A, B, C and D if every letter should appear at least once?