Reading assignment: Read carefully Section 1.3 as well as Section 2.1, to the end of page 27; glance at the rest of Section 2.1.

Written assignment: Solve the following problems. Please don't forget to follow the Homework Guidelines (see our web-page).

1. Prove that if a, b, and c are real numbers such that $a < b < c$, then $|b| < \max\{|a|, |c|\}$.

2. Prove each of the following:

   (a) If $c < 0$ and $|x - c| < \frac{|c|}{2}$, then $x < \frac{c}{2}$.

   (b) If $|x - 1| < \frac{1}{2}$ and $|y - 1| < \frac{1}{2}$, then $|x - y| < 1$.

   (c) There is no real number x such that $|x - 1| < \frac{1}{2}$ and $|x - 2| < \frac{1}{2}$.

3. Let a and b be real numbers.

   (a) Prove that $|a| - |b| \leq |a + b|$ and $|b| - |a| \leq |a + b|$.

   (b) Use part (a) to show that $\big||a| - |b|\big| \leq |a + b|$.

   (c) Use part (b) to show that $\big||a| - |b|\big| \leq |a - b|$. (This is known as the Reverse Triangle Inequality.)

4. Using only the Axioms and Elementary Properties of the real numbers, prove Cauchy's Inequality: for all real x and y, $xy \leq \frac{1}{2}(x^2 + y^2)$.

5. Let x and y be non-negative real numbers and let n be a natural number. Use the Difference of Powers identity to prove the following

   (a) If $x \leq y$, then $x^n \leq y^n$.

   (b) If $x \leq y$, then $x^{1/n} \leq y^{1/n}$.

   (c) If $x \geq y$, then $x^n - y^n \geq (x - y)ny^{n-1}$.

6. Let x be a non-negative real number and let n be a natural number. Use the Binomial Formula to prove the following.

   (a) $(1 + x)^n \geq 1 + nx$. (This is Bernoulli's Inequality.)

   (b) $(1 + x)^n \geq 1 + nx + \frac{n(n-1)}{2}x^2$.

7. Determine whether each statement is true or false. If the statement is true, prove it. If the statement is false, provide a counter-example or other justification.

   (a) For all real numbers a and b, $|a + b| \leq |a| - |b|$.

   (b) If $\{|a_n|\}$ converges, then $\{a_n\}$ converges.

   (c) If $\big\{(a_n)^2\big\}$ converges, then $\{a_n\}$ converges.