1. Prove that if $a$, $b$, and $c$ are real numbers such that $a < b < c$, then $|b| < \max\{|a|, |c|\}$.

2. Prove each of the following:
   (a) If $c < 0$ and $|x - c| < \frac{|c|}{2}$, then $x < \frac{c}{2}$.
   (b) If $|x - 1| < \frac{1}{2}$ and $|y - 1| < \frac{1}{2}$, then $|x - y| < 1$.
   (c) There is no real number $x$ such that $|x - 1| < \frac{1}{2}$ and $|x - 2| < \frac{1}{2}$.

3. Let $a$ and $b$ be real numbers.
   (a) Prove that $|a| - |b| \leq |a + b|$ and $|b| - |a| \leq |a + b|$.
   (b) Use part (a) to show that $|a| - |b| \leq |a + b|$.
   (c) Use part (b) to show that $|a| - |b| \leq |a - b|$. (This is known as the Reverse Triangle Inequality.)

4. Using only the Axioms and Elementary Properties of the real numbers, prove Cauchy’s Inequality: for all real $x$ and $y$, $xy \leq \frac{1}{2}(x^2 + y^2)$.

5. Let $x$ and $y$ be non-negative real numbers and let $n$ be a natural number. Use the Difference of Powers identity to prove the following
   (a) If $x \leq y$, then $x^n \leq y^n$.
   (b) If $x \leq y$, then $x^{1/n} \leq y^{1/n}$.
   (c) If $x \geq y$, then $x^n - y^n \geq (x - y)ny^{n-1}$.

6. Let $x$ be a non-negative real number and let $n$ be a natural number. Use the Binomial Formula to prove the following.
   (a) $(1 + x)^n \geq 1 + nx$. (This is Bernoulli’s Inequality.)
   (b) $(1 + x)^n \geq 1 + nx + \frac{n(n-1)}{2}x^2$.

7. Determine whether each statement is true or false. If the statement is true, prove it. If the statement is false, provide a counter-example or other justification.
   (a) For all real numbers $a$ and $b$, $|a + b| \leq |a| - |b|$.
   (b) If $\{|a_n|\}$ converges, then $\{a_n\}$ converges.
   (c) If $\{(a_n)^2\}$ converges, then $\{a_n\}$ converges.