Graded Homework #1: Collected Wednesday 04/09

1. Which of the following expressions are statements? Circle the statements.
   a) The US has 49 states.
   b) I like to eat pizza and you hate going to the movies.
   c) Call me on Thursday if you are home.
   d) If we go out tonight, we’ll be tired tomorrow.
   e) 3<4
   f) 4+3
   g) If $x>2$ then $x^3>1$.

2. Let $P=\text{"I am happy"}, \ Q=\text{"I am watching a movie"}, \ R=\text{"I am studying for Math310"}$.
   a) Translate the following symbolic statements into equivalent English sentences:
      i. not (P or Q)
      ii. $Q \Rightarrow \text{not } R$
      iii. (not P and R) or Q
   b) Translate the following statements from English into symbolic statements:
      i. I am neither studying for Math 310, nor watching a movie.
      ii. I am happy when I study for Math 310.
      iii. I don’t study for Math310 if I am watching a movie.
   c) If P and R are true, but Q is false, what are the truth values of the six statements from parts a) and b)?

3. Fill in the truth tables for the following statements:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>((P=&gt;Q) and P )=&gt; Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
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<td>T</td>
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<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Not (P=&gt;Q) =&gt; P</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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4. a) Give definitions for
   i. A logical tautology (hint: statements such as those in problem 2.4)
   ii. The contrapositive of the implication P=>Q.
   b) Is an implication logically equivalent to its counterpositive or not? Justify your answer.
   c) Which of the following statements are tautologies?
      i. If John eats an anchovy pizza, then he either eats an anchovy pizza or he does not.
      ii. If John either eats an anchovy pizza or he does not, then he eats an anchovy pizza.
      iii. If pigs have wings and pigs do not have wings, then the sun sets in the East.
   d) State i. the converse & ii. the contrapositive of the statement: If it’s Tuesday, we must be in Belgium.

5. Prove this Lemma: Any odd integer $n$ can be written as $n = 2k + 1$ for some integer $k$.
   (Recall: An odd integer was defined as an integer which is not even, i.e. it is not divisible by 2)

6. Prove that for any integer $n$, $n^2 + n$ is even.
   (Recall: an integer $x$ is even iff there exists an integer $k$ such that $x=2k$.)

7. Prove that for all non-negative real numbers $x$ and $y$, $\frac{x+y}{2} \geq \sqrt{xy}$.
   In your proof, explicitly indicate where you use the assumption that $x$ and $y$ are non-negative.