Math 310: Homework 7 (Ch 14) – due Wednesday, 11/26

In pbls 3 and 4 below, you may assume that the Continuum Hypothesis holds.

1. Use Cantor’s diagonal argument to write a complete formal proof showing that the interval of real numbers [2,3] is uncountable.

2. Prove that if \( A \) and \( B \) are infinite countable sets, then \( A \cup B \) is also countable. You may assume \( A \) and \( B \) are disjoint.

3. Determine the cardinality of the following sets. Your answer should be either an integer number, or one of \( \aleph_0 \), \( \aleph_1 \), \( \aleph_2 \), etc. No proof is needed (but you should be able to justify your answer if asked!).
   a) the irrational numbers
   b) \( \mathbb{Q} \times \mathbb{Q} \)
   c) \( S = \{ \sqrt{n} \mid n \in \mathbb{Q}^2 \} \)
   d) \( T = \{ \sqrt[n]{m} \mid n \in \mathbb{N}, m \in \mathbb{N} \} \)
   e) \( A = \{ n \in \mathbb{Z} \mid 0 \leq n \leq 41 \} \)
   f) the power set of the rational numbers, \( \mathcal{P}(\mathbb{Q}) \)
   g) the complex numbers: \( \mathbb{C} = \{ x + iy \mid x, y \in \mathbb{R}, i = \sqrt{-1} \} \)

4. Give examples of sets (other than precisely \( \mathbb{N}_{81}, \mathbb{N}, \mathbb{Z}, \mathbb{Q} \) or \( \mathbb{R} \)) with cardinality:
   a) 81
   b) 0
   c) \( \aleph_1 \)
   d) \( \aleph_5 \)