Math 300 -- **SETS** Worksheet (Ch 6)

<u>RELATIONSHIPS:</u>

- Inclusion (subset) $\boldsymbol{A \subseteq B} \iff [\boldsymbol{x \in A} \Rightarrow \boldsymbol{x} \in \boldsymbol{B}]$
- Equality $\boldsymbol{A = B} \iff [\boldsymbol{A \subseteq B}\ \boldsymbol{and}\ \boldsymbol{A \supseteq B}]$ (double inclusion)

  $\iff [\boldsymbol{x \in A} \iff \boldsymbol{x} \in \boldsymbol{B}]$
- Proper Subset $\boldsymbol{A \subset B} \iff \boldsymbol{A \subseteq B}$ **and** $\boldsymbol{A \neq B}$

  $\iff [(\boldsymbol{x \in A} => \boldsymbol{x} \in \boldsymbol{B})$ **and** (there exists some $\boldsymbol{a} \in \boldsymbol{B} - \boldsymbol{A}$)]

<u>OPERATIONS on Sets (new sets from old):</u>

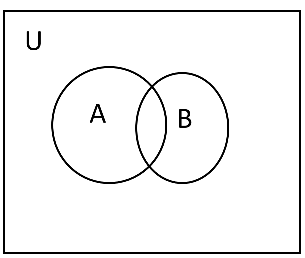
- Set Intersection: $\boldsymbol{A \cap B} = \{\boldsymbol{x}\,|\boldsymbol{x} \in \boldsymbol{A}$ **and** $\boldsymbol{x} \in \boldsymbol{B}\}$
- Set Union: $\boldsymbol{A \cup B} = \{\boldsymbol{x}\,|\boldsymbol{x} \in \boldsymbol{A}$ **or** $\boldsymbol{x} \in \boldsymbol{B}\}$
- Set Difference: $\boldsymbol{A - B} = \{\boldsymbol{x}\,|\boldsymbol{x} \in \boldsymbol{A}$ **and** $\boldsymbol{x} \notin \boldsymbol{B}\}$
- Set Complement: $\boldsymbol{A^c} = \boldsymbol{U} - \boldsymbol{A} = \{\boldsymbol{x} \in \boldsymbol{U}\,|\boldsymbol{x} \notin \boldsymbol{A}\}$ (U is some universal set containing A)
- POWER Set: $\boldsymbol{\mathcal{P}(A)} = \{\boldsymbol{X}|\boldsymbol{X} \subseteq \boldsymbol{A}\}$. (the set whose elements are all the subsets of A)

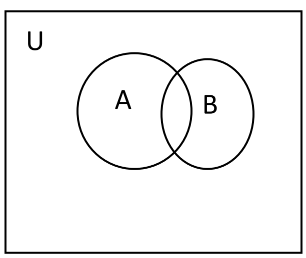
  Note that by definition of $\mathcal{P}(A)$, $\boxed{X \subseteq A <=> X \in \mathcal{P}(A)}$
- Cartesian Product: $\boldsymbol{A \times B} = \{(\boldsymbol{x}, \boldsymbol{y})|\boldsymbol{x} \in \boldsymbol{A}$ **and** $\boldsymbol{y} \in \boldsymbol{B}\}$

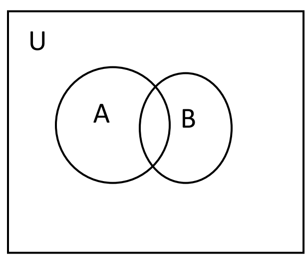
  The set of all ordered pairs (x,y), with x an element of A and y an element of B.

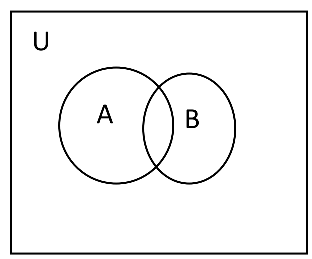
  (this is in **Ch 7**, section **7.7**)

Venn Diagrams (useful to visualize sets, but please don't use in proofs!)









<u>EMPTY SET</u>: The empty set is the set with no elements: $\boldsymbol{\phi} = \{\ \}$ (a.k.a. the <u>null set</u>)

Exercise: Prove that the empty set is a subset of any set.

Proof:

Examples:

1) Let $A = \{a, b, c, d, e\}$ and $B = \{x, y, a, z, c\}$.

$A \cap B =$

$A \cup B =$

$A - B =$

2) $(-3, 0]^c =$

3) If A={1,2,3} list all the elements of the following sets:

- $\mathcal{P}(A) =$

- $A \times A =$

<u>It's important to distinguish between subsets and elements, and to use the correct notation!</u>

Example: Suppose A={1,2,3} . Which of the following are correct statements?

$1 = \{1\}$, $1 \in A$, $1 \subseteq A$, $\{1\} \subseteq A$, $\{1,2\} \subseteq A$, $\{1,2\} \in A$,

$\{1,2\} \in \mathcal{P}(A)$, $\{1,2\} \nsubseteq \mathcal{P}(A)$, $\{\{1,2\}\} \in \mathcal{P}(A)$, $\{\{1,2\}\} \subseteq \mathcal{P}(A)$

You should recall the following results, and be able to prove them:

Theorem 6.3.4

(i) **Associativity of set union and intersection:**

$\boldsymbol{A \cup (B \cup C) =}$ $\boldsymbol{A \cap (B \cap C) =}$

(ii) **Commutativity:** $\boldsymbol{A \cup B =}$ $\boldsymbol{A \cap B =}$

(iii) **Distributivity:** $\boldsymbol{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$ **,** $\boldsymbol{A \cap (B \cup C) =}$

(iv) **De Morgan Laws:** $\boldsymbol{(A \cup B)^c =}$ $\boldsymbol{(A \cap B)^c =}$

(v) **Complementation:** $\boldsymbol{A \cup A^c =}$ $\boldsymbol{A \cap A^c =}$

(vi) **Double complement:** $\boldsymbol{(A^c)^c =}$

Sample Proofs:

Use logical arguments from the definitions to show: (iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof:

➔ You try: prove the counterpart statement: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Use truth tables to prove: (iv) $(A \cup B)^c = A^c \cap B^c$

Proposition 6.2.4: $\boldsymbol{A \cup B = (A \cap B) \cup (A - B) \cup (B - A)}$

➔ Prove it by truth tables (in text) and by logical arguments (exercise)