relations:

- Inclusion (subset) \( A \subseteq B \iff [x \in A \Rightarrow x \in B] \)
- Equality \( A = B \iff [A \subseteq B \text{ and } A \supseteq B] \) (double inclusion)
- Proper Subset \( A \subset B \iff A \subseteq B \text{ and } A \neq B \)

operations on sets (new sets from old):

- Set Intersection: \( A \cap B = \{x | x \in A \text{ and } x \in B\} \)
- Set Union: \( A \cup B = \{x | x \in A \text{ or } x \in B\} \)
- Set Difference: \( A - B = \{x | x \in A \text{ and } x \notin B\} \)
- Set Complement: \( A^c = U - A = \{x | x \in U \text{ or } x \notin A\} \) \((U \text{ is some universal set containing A)}\)
- Power Set: \( P(A) = \{X | X \subseteq A\}. \text{ (the set whose elements are all the subsets of A)}\)
- Cartesian Product: \( A \times B = \{(x, y) | x \in A \text{ and } y \in B\} \)

Venn Diagrams (useful to visualize sets, but please don’t use in proofs!)

EMPTY SET: The empty set is the set with no elements: \( \emptyset = \{\} \) (a.k.a. the null set)

Exercise: Prove that the empty set is a subset of any set.

Proof:
Examples:

1) Let $A = \{a, b, c, d, e\}$ and $B = \{x, y, a, z, c\}$.

\[
A \cap B =
\]
\[
A \cup B =
\]
\[
A - B =
\]

2) $\ (-3,0]^c =$

3) If $A = \{1,2,3\}$ list all the elements of the following sets:

- \(\mathcal{P}(A) = \)
- \(A \times A = \)

It's important to distinguish between subsets and elements, and to use the correct notation!

Example: Suppose $A = \{1,2,3\}$. Which of the following are correct statements?

\[
1 = \{1\}, \quad 1 \in A, \quad 1 \subseteq A, \quad \{1\} \subseteq A, \quad \{1,2\} \subseteq A, \quad \{1,2\} \in A,
\]
\[
\{1,2\} \in \mathcal{P}(A), \quad \{1,2\} \notin \mathcal{P}(A), \quad \{\{1,2\}\} \in \mathcal{P}(A), \quad \{\{1,2\}\} \subseteq \mathcal{P}(A)
\]

You should recall the following results, and be able to prove them:

**Theorem 6.3.4**

(i) **Associativity of set union and intersection:**

\[
A \cup (B \cup C) = \quad A \cap (B \cap C) =
\]

(ii) **Commutativity:**

\[
A \cup B = \quad A \cap B =
\]

(iii) **Distributivity:**

\[
A \cup (B \cap C) = (A \cup B) \cap (A \cup C), \quad A \cap (B \cup C) =
\]

(iv) **De Morgan Laws:**

\[
(A \cup B)^c = \quad (A \cap B)^c =
\]

(v) **Complementation:**

\[
A \cup A^c = \quad A \cap A^c =
\]

(vi) **Double complement:**

\[
(A^c)^c =
\]
Sample Proofs:

Use logical arguments from the definitions to show: (iii) \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)

Proof:

\[ \begin{array}{cccccccc}
& & & & & & & \\
& & & & & & & \\
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\end{array} \]

\( \Rightarrow \) You try: prove the counterpart statement: \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \)

Use truth tables to prove: (iv) \( (A \cup B)^c = A^c \cap B^c \)

Proposition 6.2.4: \( A \cup B = (A \cap B) \cup (A - B) \cup (B - A) \)

\( \Rightarrow \) Prove it by truth tables (in text) and by logical arguments (exercise)