Math 126 1st Midterm Review  Autumn 2004
Answers to Even Numbered Problems

Sequences – Convergence – Monotonic – Bounded
p.710 #17,19,25,56
#56: Sequence is increasing because \( f'(x) = \frac{17}{(3x+4)^2} > 0 \) for \( x = n \). It is bounded below by \( \frac{1}{7} \) and above by \( \frac{2}{3} \).

Series – Partial Sums – Convergence – Geometric Series – Test for Divergence
p.720 #11,24,27,36
#24 Series diverges, by the Divergence Test, since \( \lim a_n = 1 \neq 0 \).
#36 \( 0.73 = \frac{73/10^8}{1-1/10^2} = \frac{73}{99} \)

\( p \)-series
p.729 #3,4,9
#4: series is divergent since it’s a \( p \)-series with \( p = 1/4 < 1 \).
#3: convergent (\( p = 4 > 1 \)), #9 divergent (\( p = .85 < 1 \))

Alternating Series Theorem – Error Estimate
p.739 #8,10,11,14,23,30
#8: Show \( b_n \) is decreasing and \( b_n \) converges to zero, so by the Alternating Series Test the series is convergent.
#10 \( \lim b_n = 1/2 \), so \( \lim a_n \) does not exist (hence \( \neq 0 \)). By the Divergence Test, the series diverges.
#14: Alternating series. Since \( f'(x) = \frac{1-lnx}{x^2} < 0 \) for \( x > e \), the sequence is monotonically decreasing for \( n \) large enough. Also show \( \lim_{n \to \infty} b_n = 0 \). By the AST, the series converges.
#30: \( b_6 \approx .0000019 \), so approximate the series with the partial sum for \( n = 1 \) to 5 to get series \( \approx -.283471 \) correct to 4 decimal digits.

Ratio Test – Root Test – Absolute Convergence – Conditional Convergence
p.745 #7,9,14,20,28
#14 Apply Ratio Test to get \( \lim = 0 < 1 \). Hence the series is absolutely convergent.
#20: Apply Root Test to get \( \lim = 0 < 1 \). Hence the series is absolutely convergent.
#28: Ratio Test gives \( \lim = 2/3 < 1 \), so the series is absolutely convergent.

Power Series – Radius of Convergence – Interval of Convergence
p.753 #4,11,14,18,23,29
#4 Ratio Test: \(|x| < 1\) (radius 1 away from zero). Interval: \((-1, 1)\), i.e diverges at \(x=-1\) and converges (conditionally) at \(x=1\) (use AST and p-series)
#14: Ratio test: \(\lim x = 0\) for all \(x\), so \(R = \infty\) and the interval is \((-\infty, \infty)\).
#18: Ratio test: \(|2x + 3| < 1\) so \(R = 1/2\) away from -3. Checking endpoints and using p-series and AST get convergence interval \((-7/2, -5/2)\).

Functions and Power Series – Term-by-term Differentiation and Integration
p.759 #4,6,12,16,18,24,30
#4: \(\sum_{n=0}^{\infty} 3x^{4n}, R = 1\) away from zero.
#6: \(\sum_{n=0}^{\infty} (-1)^{n}3^{2n}x^{2n}, R = \frac{1}{3}\) from zero.
#12: \(\sum_{n=0}^{\infty} [2(-1)^{n} - 3]^nx^n, R = \frac{1}{3}\) from zero.
#16: \(\sum_{n=0}^{\infty} 2^n(n+1)x^{n+2} = \sum_{n=2}^{\infty} 2^{n-2}(n-1)x^n, R = \frac{1}{2}\) from zero.
#18: \(\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{3^{2n+1}(2n+1)^2}, R = 3\) from zero.
#24: \(-\sum_{n=1}^{\infty} \frac{n}{n^2} + C, R = 1\) from zero. Can use Example 6 to get power series for \(\ln(1-t)\).
#30: \(\sum_{n=0}^{\infty} (-1)^n \frac{1}{n(6n+1)2^{6n+1}}.\) Alternating Series. Using the first 3 terms \((n = 0\) to \(n = 2)\), the errors is no larger than \(|a_3| = \frac{1}{19 \times 2^{19}} \approx 10^{-7}\). To six decimal places, the integral is 0.498893.

Taylor and Maclaurin Series – Remainder and Taylor’s Inequality
p.770 #4,17,27,28,40,45,56
#4: \(\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}x^{2n+1}}{(2n+1)!}, R = \infty\) from zero.
#28 \(\sum_{n=0}^{\infty} \frac{(-1)^n2^n}{(2n)!} x^{2n+1}, R = \infty\) from zero (see series for cos and radius, from table).
#40: \(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} + C\).
#56: \(\sqrt{\frac{3}{2}}\).

Taylor Polynomials – Applications
p.783 #18(a),26,28
#18(a): \(f(x) \approx T_3(x) = \ln 3 + \frac{2}{3}(x-1) - \frac{4}{92!}(x-1)^2 + \frac{16/27}{3!}(x-1)^3\).
#26: we need the first five non-zero terms of the series, by AST and comparing terms to desired accuracy.
#28: \(|\text{error}| < | - x^6/6!| < 0.005\) i.e. \(|x| < (3.6)^{1/6}\).