Sample Midterm 1

This page contains problems similar to those you will see on the midterm exam. It is supposed to help you review. Justify all your answers.

1. Do the following series converge or diverge?

   (a) \[ \sum_{n=0}^{\infty} (-1)^n \frac{n^2 + 3}{5n^2 - n} \]

   (b) \[ \sum_{n=0}^{\infty} (-1)^n \frac{n + 1}{5n^2 - n} \]

   (c) \[ \sum_{k=0}^{\infty} \frac{5^k}{3^k + 4^k} \]

2. Find the interval of convergence for \[ \sum_{n=0}^{\infty} \frac{(x + 2)^n}{n4^n} \]. Leave your answer as an open interval, i.e. don’t check the convergence at endpoints.

3. Suppose that \[ \sum_{n=0}^{\infty} c_n (x + 1)^n \] converges when \( x = 2 \) and diverges when \( x = -5 \). Can you say if \[ \sum_{n=0}^{\infty} (-1)^n c_n \] converges? How about \[ \sum_{n=0}^{\infty} (-3)^n c_n \]?

4. Find the power series centered at zero for the function \( f(x) = \frac{x}{2 + x^3} \). What is the radius of convergence?

5. Compute the Taylor polynomial \( T_3(x) \) for \( g(x) = \sqrt{4 + x} \), with \( a = 5 \).