1a). Compute indefinite integral: \[ \int x^2 \sin x^3 \, dx \]

Simple substitution:

\[ u = x^3 \quad \Rightarrow \quad du = 3x^2 \, dx \quad \Rightarrow \quad \frac{du}{3} = x^2 \, dx \]

The integral transforms to

\[ \int x^2 \sin x^3 \, dx = \int \frac{\sin u}{3} \, du = \frac{1}{3} (-\cos u) + C \]

Resubstitute for \( x \)

\[ \int x^2 \sin x^3 \, dx = -\frac{1}{3} \cos (x^3) + C \]
1b) Compute the indefinite integral \( \int \ln\left(p^2 - 2p\right)dp \)

Simplify first:

\[
\int \ln\left(p^2 - 2p\right)dp = \int \ln\left(p\left(p - 2\right)\right)dp = \int \ln\left(p\right) + \ln\left(p - 2\right)dp
\]

Integration by parts for first term

\[
u = \ln p \quad dv = 1dp
\]

\[
du = \frac{1}{p}dp \quad v = p
\]

Transforms the first term of the integral to

\[
\int \ln\left(p\right)dp = p \ln\left(p\right) - \int p \cdot \frac{1}{p}dp = p \ln\left(p\right) - \int dp
\]

\[
= p \ln\left(p\right) - p
\]

For the second term, do a simple substitution first

\[
x = p - 2 \quad \Rightarrow \quad dx = dp
\]

Which transforms the second term to

\[
\int \ln\left(p - 2\right)dp = \int \ln\left(x\right)dx
\]

Using the results obtains above, and resubstituting we obtain

\[
\int \ln\left(p - 2\right)dp = x \ln\left(x\right) - x
\]

\[
= (p - 2) \ln\left(p - 2\right) - (p - 2)
\]
Combine the two terms to obtain the anti-derivative and simplify

\[
\int \ln(p^2 - 2p) \, dp = \left[ p \ln(p) - p \right] + \left[ (p - 2) \ln(p - 2) - (p - 2) \right] + C \\
= p \ln(p) + \ln(p - 2) - 2 \ln(p - 2) - 2p + 2 + C \\
= p \ln(p^2 - 2p) - 2 \ln(p - 2) - 2p + \tilde{C} \\
= p \ln(p^2 - 2p) - 2 \ln(p - 2) - 2p + \tilde{C}
\]

The final answer is:

\[
\int \ln(p^2 - 2p) \, dp = p \ln(p^2 - 2p) - 2 \ln(p - 2) - 2p + \tilde{C}
\]

Note that the constant term, 2, was “absorbed” into the arbitrary constant.
2a. Evaluate \( I = \int_{0}^{3} \frac{x^3 + 6}{x^2 + 3x + 2} \, dx \)

Recognize this as a rational function, so we need to do partial fractions. The degree of the numerator (3) is greater than (or equal to) the degree of the denominator (2), so first do long division of polynomials:

\[
\frac{x - 3}{x^2 + 3x + 2} \Rightarrow \frac{x^3 + 6}{x^2 + 3x + 2} = x - 3 + \frac{7x + 12}{x^2 + 3x + 2}
\]

Now do partial fraction decomposition on the remainder term:

\[
\frac{7x + 12}{x^2 + 3x + 2} = \frac{7x + 12}{(x + 2)(x + 1)} = \frac{A}{x + 2} + \frac{B}{x + 1} \Rightarrow A(x + 1) + B(x + 2) = 7x + 12
\]

\[
x = -1 \Rightarrow A(0) + B(-1) = 7(-1) + 12 \Rightarrow B = 5
\]

\[
x = -2 \Rightarrow A(-1) + B(0) = 7(-2) + 12 \Rightarrow A = 2
\]
So

\[
\int_0^3 \frac{x^3 + 6}{x^2 + 3x + 2} \, dx = \int_0^3 x - 3 + \frac{2}{x + 2} + \frac{5}{x + 1} \, dx
\]

\[
= \left[ \frac{x^2}{2} - 3x + 2 \ln(x + 2) + 5 \ln(x + 1) \right]_0^3
\]

\[
= \frac{3^2}{2} - 3 \cdot 3 + 2 \ln(5) + 5 \ln(4) - 2 \ln(2) - 5 \ln(1)
\]

\[
= -\frac{3^2}{2} + 2 \ln(5) + 10 \ln(2) - 2 \ln(2)
\]

\[
= -\frac{9}{2} + 2 \ln(5) + 8 \ln(2)
\]

The final answer is

\[
\int_0^3 \frac{x^3 + 6}{x^2 + 3x + 2} \, dx = -\frac{9}{2} + 2 \ln(5) + 8 \ln(2)
\]
2b. Evaluate \( \int_{2}^{4} \frac{dx}{2x^2\sqrt{x^2-3}} \).

Use an Inverse trig substitution:

\[
x = \sqrt{3}\sec\theta \\
dx = \sqrt{3}\sec\theta \tan\theta \, d\theta
\]

\[
\sqrt{x^2-3} = \sqrt{3}\sec^2\theta - 3 = \sqrt{3}\sec^2\theta - 1 = \sqrt{3}\tan^2\theta = \sqrt{3}\tan\theta
\]

The integral transforms to

\[
\int_{2}^{4} \frac{dx}{2x^2\sqrt{x^2-3}} = \int \frac{\sqrt{3}\sec\theta \tan\theta \, d\theta}{2 (\sqrt{3}\sec\theta)^2 \sqrt{3}\tan\theta} = \frac{1}{6} \int \cos\theta \, d\theta = \frac{1}{6} \sin\theta
\]

Use the picture to obtain the relationship for re-substitution

\[
\sin\theta = \frac{\sqrt{x^2-3}}{x}
\]

So

\[
\int_{2}^{4} \frac{dx}{2x^2\sqrt{x^2-3}} = \left[ \frac{1}{6} \frac{\sqrt{x^2-3}}{x} \right]_{2}^{4} = \frac{1}{6} \left[ \frac{\sqrt{4^2-3}}{4} - \frac{\sqrt{2^2-3}}{2} \right] = \frac{\sqrt{13}}{24} - \frac{1}{12}
\]
3. A solid is obtained by rotating the area shown about the \( y \)-axis. The curve that forms the inside of solid is the graph of \( y = e^{x^2} \). Express the volume of the solid as an integral formula. SETUP ONLY. DO NOT EVALUATE THE INTEGRALS.

By Disks/Washers:
We must split the region up into two:

For Region 1:
\[
\text{Radius of Slice: } r_i = x = 4 - y \\
\text{Area of Slice: } A_i = \pi r_i^2 = \pi (4 - y_i)^2 \\
\text{Volume of Slice: } V_i = \pi (4 - y_i)^2 \Delta y \\
\text{Volume of Region: } V = \int_{y=0}^{y=e^{-2}} \pi (4 - y_i)^2 \, dy
\]
For Region 2 (washers):

Outer Radius of Slice: \( r_{outer} = 4 - y \)

Inner Radius of Slice: \( r_{inner} = \ln y + 2 \)

Area of Slice: \( A_i = \pi r_{outer}^2 - \pi r_{inner}^2 \)

\[ = \pi \left[ (4 - y_i)^2 - (\ln y + 2)^2 \right] \]

Volume of Slice: \( V_i = \pi \left[ (4 - y_i)^2 - (\ln y + 2)^2 \right] \Delta y \)

Volume of Region 2: \( V = \int_{y=e^{-2}}^{y=1} \pi \left[ (4 - y_i)^2 - (\ln y + 2)^2 \right] dy \)

Add both regions together to obtain the total volume of revolution:

\[
V = \int_{y=0}^{y=1} \pi (4 - y_i)^2 dy + \int_{y=e^{-2}}^{y=1} \pi \left[ (4 - y_i)^2 - (\ln y + 2)^2 \right] dy
\]

\[= \pi \int_{y=0}^{y=1} (4 - y_i)^2 dy - \pi \int_{y=e^{-2}}^{y=1} (\ln y + 2)^2 dy\]

Which can be interpreted as a difference of the volumes of revolution between the line and the exponential.
Method 2: Cylindrical Shells. Although the Disk/Washer method led to only two integrals, in practice, the shell method leads to three “easier” integrals.

The region must be split into 3 regions and integrals. For each region we pick a perpendicular “slice” and compute surface area of cylindrical shell of radius \( r \) and height \( h \):

Region 1: \( 0 \leq x \leq 2 \): \( r = x, \ h_1 = e^{x-2} \)
Region 2: \( 2 \leq x \leq 3 \): \( r = x, \ h_2 = 1 \)
Region 3: \( 3 \leq x \leq 4 \): \( r = x, \ h_3 = 4 - x \)

Each cylindrical slice has area: \( A = 2\pi rh \)

The volume of each region is obtained by integrating the cylinder areas across the region:

\[
V_1 = 2\pi \int_0^2 xe^{x-2} \, dx, \quad V_2 = 2\pi \int_2^3 x \, dx, \quad V_3 = 2\pi \int_3^4 x \left(4 - x\right) \, dx,
\]

The total volume is the sum of the volumes of the three regions

\[
V = V_1 + V_2 + V_3 = 2\pi \left[ \int_0^2 xe^{x-2} \, dx + \int_2^3 x \, dx + \int_3^4 x \left(4 - x\right) \, dx \right]
\]
4. A water tank is completely filled with water (specific weight 1000kg/m$^3$). Front and back are 1.5m apart and have the shape of a trapezoid (width at bottom 1m, at top 2m, and height 1m), see picture. The outlet of the water tank is 0.25m higher than the top of the tank. Set up an integral for the work required to pump all the water out of the tank through the outlet. Recall that the acceleration due to gravity is $g = 9.8\text{m/s}^2$. SET UP ONLY. DO NOT EVALUATE THE INTEGRAL.

Choose origin to be at lower left corner of tank, and consider a “slice” at height $y_i$ from the bottom. The width of the slice is clearly linearly related to $y$ and we require

$$w(0) = 1, \quad w(1) = 2 \quad \Rightarrow \quad w(y_i) = 1 + y_i$$

Thus

$$Area : A_i = (1.5) w(y_i) = \frac{3}{2} (1 + y_i)$$

$$Volume : V_i = A_i \Delta y = \frac{3}{2} (1 + y_i) \Delta y$$
To calculate work, we need a FORCE not a volume, so we convert the volume to a mass and then a weight via:

\[ \text{Mass: } m_i = \rho V_i = \frac{3}{2} \rho (1 + y_i) \Delta y, \quad (\rho = 1000 \text{kg/m}^3) \]

\[ \text{Force: } F_i = m_i a = \frac{3}{2} \rho g (1 + y_i) \Delta y, \quad (g = 9.8 \text{m/s}^2) \]

The distance required to move the slice (including the outlet) is:

\[ \text{Distance: } d_i = 1 - y_i + 0.25 = \frac{5}{4} - y_i \]

Since \( \text{Work} = \text{Force} \times \text{Distance} \), the work to pump the slice to the outlet is

\[ W_i = F_i d_i = \frac{3}{2} \rho g (1 + y_i) \left( \frac{5}{4} - y_i \right) \Delta y. \]

Obtain the total work by “adding up” all the slices via integration:

\[ \text{Total Work, } W = \frac{3}{2} \rho g \int_{0}^{1} (1 + y) \left( \frac{5}{4} - y \right) dy \]

\[ = \frac{3}{2} (9.8)(1000) \int_{0}^{1} (1 + y) \left( \frac{5}{4} - y \right) dy \]

\[ = 14,700 \int_{0}^{1} (1 + y) \left( \frac{5}{4} - y \right) dy \]
5. Find the solution to the differential equation \( \frac{dz}{dt} = (4 + z^2) t \) with initial condition \( z(0) = 2 \).

Separate variables treating \( dz \) and \( dt \) as variables:

\[
\frac{dz}{4 + z^2} = t \, dt
\]

Integrate both sides:

\[
\int \frac{dz}{4 + z^2} = \int t \, dt
\]

For the left hand side

\[
\int \frac{dz}{4 + z^2} = \int \frac{dz}{4(1 + z^2/4)} = \frac{1}{4} \int \frac{dz}{1 + (z/2)^2}
\]

Let \( u = z/2 \), \( \Rightarrow \) \( du = dz/2 \) \( \Rightarrow \) \( 2 \, du = dz \)

\[
\int \frac{dz}{4 + z^2} = \frac{1}{4} \int \frac{2 \, du}{1 + u^2} = \frac{1}{2} \arctan u = \frac{1}{2} \arctan \left( \frac{z}{2} \right) + C_1
\]

For the right hand side

\[
\int t \, dt = \frac{t^2}{2} + C_2
\]

Thus

\[
\frac{1}{2} \arctan \left( \frac{z}{2} \right) + C_1 = \frac{t^2}{2} + C_2
\]
Solving for \( z \), we obtain

\[
\arctan \left( \frac{z}{2} \right) = t^2 + (C_2 - 2C_1)
\]

\[
z = 2 \tan \left( t^2 + (C_2 - 2C_1) \right)
\]

The two arbitrary constants may be combined into a single constant so that

\[
z(t) = 2 \tan \left( t^2 + C \right), \quad \left( C = (C_2 - 2C_1) \right)
\]

Now apply the initial condition and solve for \( C \)

\[
z(0) = 2 \tan \left( 0^2 + C \right) = 2 \tan(C) = 2 \quad \Rightarrow \quad \tan(C) = 1
\]

Thus

\[
C = \frac{\pi}{4} + k\pi = \pi \left( k + \frac{1}{4} \right), \quad (k \in \mathbb{Z})
\]

and so the solution to the initial value problem is

\[
z(t) = 2 \tan \left( t^2 + \pi \left( \frac{1}{4} + k \right) \right), \quad (k \in \mathbb{Z})
\]
6. Let \( g(y) = 2y^2 - 5y + 2 \). Set up an integral for the length of the parabola \( x = g(y) \) between the points \((-1,1)\) and \((0,2)\). SET UP ONLY. DO NOT EVALUATE THE INTEGRAL.

Don’t be confused by the function being given in \( y \) instead of \( x \). If you try to solve with \( x \) as the independent variable you will have a much harder problem!

\[
L = \int_{1}^{2} \sqrt{1 + \left( \frac{dg}{dy} \right)^2} \, dy
\]

The derivative is

\[
\frac{dg}{dy} = \frac{d}{dy}(2y^2 - 5y + 2) = 4y - 5
\]

\[
1 + \left( \frac{dg}{dy} \right)^2 = 1 + (4y - 5)^2 = 1 + 16y^2 - 40y + 25
\]

\[
= 16y^2 - 40y + 26
\]

Thus

\[
L = \int_{1}^{2} \sqrt{16y^2 - 40y + 26} \, dy
\]
7. A bathtub contains 300 liters of bathing water in which 0.05kg of salt are dissolved. Pure water is added to the bathtub at a rate of 10 liters per minute. The thoroughly mixed water drains from the tub at the same rate. How long does it take until half the salt has left the tub?

Let $y(t)$ be the amount of salt (in kg) in the tub at time $t$ (in minutes).

From the statement of the problem, we know the initial condition:

$$y(0) = 0.05kg$$

The basic law of mixing is

$$\frac{dy}{dt} = \left[\text{rate in}\right] - \left[\text{rate out}\right]$$

but there is NO salt coming in, so

$$\left[\text{rate in}\right] = 0$$

The rate at which liquid leaves the bathtub is specified as the same as the rate at which pure water is entering: 10 liters / minute. The concentration of salt at time $t$ is

$$\left[\text{concentration}(t)\right] = \frac{\text{total salt}(t)}{\text{total volume}} = \frac{y(t)}{300}$$

So the rate at which salt leaves the system is:

$$\left[\text{rate out}\right] = \left[\text{concentration}\right] \times \left[\frac{10 \text{ liters}}{\text{minute}}\right] = \frac{10}{300} y(t) = \frac{1}{30} y(t)$$
So the differential equation governing the amount of salt in the system is
\[
\frac{dy}{dt} = -\frac{1}{30} y, \quad y(0) = 0.05
\]
Separation of variables gives:
\[
\frac{30}{y} \, dy = - \, dt
\]
Integrating both sides we obtain
\[
\int \frac{30}{y} \, dy = \int -1 \, dt
\]
\[
30 \ln |y| = -t + C \quad \Rightarrow \quad |y| = e^{-1/30t+C} = e^C \, e^{-1/30t}
\]
\[
y(t) = Ae^{-1/30t}, \quad (A = \pm e^C)
\]
Apply the initial condition to find the constant \(A\):
\[
y(0) = Ae^0 = A = 0.05
\]
Thus, the solution to the ODE is
\[
y(t) = 0.05e^{-1/30t}
\]
We seek to find the time, \(t\) at which \(y(t) = \) half the initial concentration, or
\[
y(t) = \frac{0.05}{2} = 0.05e^{-1/30t} \quad \Rightarrow \quad \frac{1}{2} = e^{-1/30t} \quad \Rightarrow \quad \ln \left( \frac{1}{2} \right) = -t / 30
\]
\[
\Rightarrow \quad t = -30 \ln \left( \frac{1}{2} \right) = 30 \ln (2) = 20.79 \text{ minutes}
\]
8. Find the $x$-coordinate of the center of mass of the region between the curves $y = 0$ and $y = x \sin x$, and between $x = 0$ and $x = \pi$.

Recall that $\bar{x} = \frac{1}{A} \int x f(x) \, dx$, where $A$ is the area of the region. Let’s start with the area:

$$A = \int_{0}^{\pi} x \sin x \, dx$$

Requires integration by parts with

$$u = x \quad dv = \sin x \quad du = dx \quad v = -\cos x$$

so the integral transforms to

$$\int_{0}^{\pi} x \sin x \, dx = x(-\cos x) - \int (-\cos x) \, dx$$

$$= -x \cos x + \int \cos x \, dx$$

$$= [-x \cos x + \sin x]_{0}^{\pi}$$

$$= [-\pi \cos \pi + \sin \pi] - [-0 \cos 0 + \sin 0]$$

$$= [-\pi (-1) + 0] - [0] = \pi$$

So the area of the region is just

$$A = \pi$$
Now the $x$-coordinate of the center of mass is

$$\bar{x} = \frac{1}{A} \int x f(x) \, dx = \frac{1}{\pi} \int_0^\pi x^2 \sin x \, dx$$

which requires integration by parts, twice. For the first one,

$$u = x^2 \quad dv = \sin x$$
$$du = 2x \, dx \quad v = -\cos x$$

$$\frac{1}{\pi} \int_0^\pi x^2 \sin x \, dx = \frac{1}{\pi} \left[ x^2 (-\cos x) - \int 2x (-\cos x) \, dx \right]$$

$$= \frac{1}{\pi} \left[ -x^2 \cos x + \int 2x \cos x \, dx \right]$$

Now for the remaining integral, integrate by parts again with

$$u = 2x, \quad dv = \cos x$$
$$du = 2 \, dx, \quad v = \sin x$$

Thus

$$\int 2x \cos x \, dx = 2x \sin x - \int 2 \sin x \, dx$$
$$= 2x \sin x + 2 \cos x$$

And so

$$\bar{x} = \frac{1}{\pi} \int_0^\pi x^2 \sin x \, dx = \frac{1}{\pi} \left[ -x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^\pi$$
Being careful in the evaluation we obtain,

\[
\bar{x} = \frac{1}{\pi} \left[ \left( -\pi^2 \cos \pi + 2\pi \sin \pi + 2\cos \pi \right) - \left( -0^2 \cos 0 + 2(0) \sin x + 2\cos 0 \right) \right]
\]

\[
= \frac{1}{\pi} \left[ (-\pi^2 (-1) + 2\pi (0) + 2(-1)) - (0 + 0 + 2(1)) \right]
\]

\[
= \frac{1}{\pi} \left[ \pi^2 - 2 \right] = \frac{\pi^2 - 4}{\pi} \approx 1.8684
\]

Which looks about right on the figure.

Although the problem does not request the \( y \)-coordinate of the center of mass, the computation is

\[
\bar{y} = \frac{1}{A} \int_0^\pi \frac{1}{2} \left[ f(x) \right]^2 dx = \frac{1}{2\pi} \int_0^\pi x^2 \sin^2 x \, dx
\]

which can be solved by first using \( \sin^2 x = \frac{1}{2} (1 - \cos 2x) \) to obtain

\[
\bar{y} = \frac{1}{4\pi} \int_0^\pi x^2 \left( 1 - \cos (2x) \right) dx = \frac{1}{4\pi} \int_0^\pi x^2 \, dx + \frac{1}{4\pi} \int_0^\pi x^2 \cos (2x) \, dx
\]

The second integral may be solved using integration by parts twice to obtain

\[
\bar{y} = \frac{1}{4\pi} \left[ \frac{x^3}{3} - \left( \frac{1}{2} x^2 \sin 2x + \frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x \right) \right]_0^\pi = \frac{2\pi^2 - 3}{24} \approx 0.6975
\]
9. For what values of $a$ is $\int_{0}^{\infty} xe^{ax} \, dx$ convergent? Evaluate the integral where it converges.

\[
\int_{0}^{\infty} xe^{ax} \, dx = \lim_{b \to \infty} \int_{0}^{b} xe^{ax} \, dx
\]

First compute the definite integral using integration by parts with

\[
u = e^{ax}\]
\[
\frac{du}{dx} = dx,
\]
\[
\frac{dv}{dx} = e^{ax}\]

Gives

\[
\int_{0}^{b} xe^{ax} \, dx = x \frac{e^{ax}}{a} - \int_{0}^{b} \frac{e^{ax}}{a} \, dx = \left[ x \frac{e^{ax}}{a} - \frac{e^{ax}}{a^2} \right]_{0}^{b}
\]
\[
= \left[ \frac{1}{a^2} (ax-1) e^{ax} \right]_{0}^{b} = \frac{1}{a^2} \left[ (ab-1)e^{ab} - (-1)e^{0} \right]
\]
\[
= \frac{1}{a^2} \left[ (ab-1)e^{ab} + 1 \right]
\]

It is useful to note that if $a = 0$, then the integral is DIVERGENT.

\[
a = 0 \quad \Rightarrow \quad \int_{0}^{b} xe^{ax} \, dx = \int_{0}^{b} x \, dx = \frac{b^2}{2} \to \infty, \quad \text{as} \quad b \to \infty.
\]
We are asked for which values of \( a \) is the integral convergent, so
\[
\int_0^\infty xe^{ax} \, dx = \lim_{b \to \infty} \int_0^b xe^{ax} \, dx = \lim_{b \to \infty} \frac{1}{a^2} \left[ (ab - 1)e^{ab} + 1 \right]
\]
Assuming the limit exists, in other words the integral converges, then
\[
\lim_{b \to \infty} \frac{1}{a^2} \left[ (ab - 1)e^{ab} + 1 \right] = \frac{1}{a^2} \left[ \lim_{b \to \infty} (ab - 1)e^{ab} + 1 \right]
\]
and when
\[
a > 0 \quad \Rightarrow \quad \lim_{b \to \infty} (ab - 1)e^{ab} \to \infty \cdot \infty = \infty \quad (\text{diverges})
\]
\[
a < 0 \quad \Rightarrow \quad \lim_{b \to \infty} (ab - 1)e^{ab} \to -\infty \cdot 0 \quad (\text{indeterminate})
\]
The second limit may be checked with L’Hopital’s rule by writing
\[
\lim_{b \to \infty} (ab - 1)e^{ab} = \lim_{b \to \infty} \frac{ab - 1}{e^{-ab}} = \lim_{b \to \infty} \frac{d}{db} \frac{(ab - 1)}{e^{ab}} = \lim_{b \to \infty} \frac{a}{-ae^{ab}} = \lim_{b \to \infty} e^{ab} = -e^{-\infty} = 0
\]
So, the integral is convergent for \( a < 0 \), and when \( a < 0 \)
\[
\int_0^\infty xe^{ax} \, dx = \frac{1}{a^2} \left[ \lim_{b \to \infty} (ab - 1)e^{ab} + 1 \right] = \frac{1}{a^2} \left[ 0 + 1 \right] = \frac{1}{a^2}
\]
10. Find the area between the curves $y = \cos x$ and $y = \sin 2x$ for $0 \leq x \leq \pi / 2$.

The only thing to be careful of here is to pay attention to when each curve is on top, and details of evaluation.

$$A = \int_0^{\pi/6} \cos x - \sin 2x \, dx + \int_{\pi/6}^{\pi/2} \sin 2x - \cos x \, dx$$

$$A = \left[ \sin x + \frac{1}{2} \cos 2x \right]_0^{\pi/6} + \left[ -\frac{1}{2} \cos 2x - \sin x \right]_{\pi/6}^{\pi/2}$$

$$= \left[ \sin x + \frac{1}{2} \cos 2x \right]_0^{\pi/6} + \left[ \sin x + \frac{1}{2} \cos 2x \right]_{\pi/6}^{\pi/2}$$

$$= \left[ -y - x \right]_a^b = \left[ x + y \right]_a^b$$

$$= 2 \left[ \sin \frac{\pi}{6} + \frac{1}{2} \cos 2 \frac{\pi}{6} \right] - \left[ \sin \frac{\pi}{2} + \frac{1}{2} \cos 2 \frac{\pi}{2} \right] - \left[ \sin 0 + \frac{1}{2} \cos 0 \right]$$

$$= 2 \left[ \frac{1}{2} + \frac{1}{2} \right] - \left[ 1 - \frac{1}{2} \right] - \left[ \frac{1}{2} \right] = 1 + \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$$
11. Consider the function \( E(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x^2} e^{-t^2} dt \).

a). Compute \( E'(x) \)

By FTC1 and the chain rule,

\[
\frac{dE}{dx} = \frac{d}{dx} \frac{2}{\sqrt{\pi}} \int_{0}^{x^2} e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \frac{d}{du} \left( \int_{0}^{u} e^{-t^2} dt \right) \frac{du}{dx}, \quad (u(x) = x^2)
\]

\[
= \frac{2}{\sqrt{\pi}} e^{-u^2} \frac{du}{dx} = \frac{2}{\sqrt{\pi}} e^{-(x^2)^2} 2x = \frac{4x}{\sqrt{\pi}} e^{-x^4}
\]

b) For what values of \( x \) is \( E(x) \) increasing?

\( E \) is increasing when the derivative is positive or

\[ E(x) \text{ increasing} \iff E'(x) > 0 \iff \frac{4x}{\sqrt{\pi}} e^{-x^4} > 0 \]

But \( e^{-x^4} > 0 \) for every \( x \in \mathbb{R} \), so \( E \) is increasing only for \( x > 0 \).