Some practice problems for Midterm 2

1. Calculate the derivative $\frac{dy}{dx}$ for each of the following functions:
   
   a) $y = 2 \tan^2(x) - \sec^2(x)$.
   
   b) $y = \left(\frac{2\sqrt{x}}{2\sqrt{x}+1}\right)^2$
   
   c) $y = x^2 \sin^2(2x)$
   
   d) $x \ln y = y + x^3$
   
   e) $y = \frac{e^{2x}}{e^x+2}$
   
   f) $y = (\ln x)^{\cos x}$
   
   g) $y = \sin(e^x)$

2. Determine whether the curve $y = \sin(t - \sin t)$ has a horizontal tangent at the origin.

3. Find the equation of the tangent line to the following curve at the given point:
   
   $x + \sqrt{xy} = 6$, at the point (4,1).

4. Find the equation of the tangent line to $y = e^x$ that is parallel to the line $x-4y=1$.

5. A particle moves on a vertical line so that its coordinate at time $t$ seconds is:
   
   $y = t^3 - 12t + 3$ (in feet)
   
   a) What is the acceleration of this particle at time 4 seconds? Include correct units.
   
   b) During the first 10 seconds of the motion, when is the particle moving upward and when is it moving downward?

6. Suppose a piston is moving up and down and its position at time $t$ seconds is

   $s = A \cos(2\pi bt)$

   Here $A$ and $b$ are positive constants, with the value of $A$ being the amplitude of the motion, and $b$ representing the frequency (number of times the piston moves up and down each second).

   a) Find formulas for the piston’s velocity and acceleration.

   b) What is the piston’s maximum speed?

   c) What effect does doubling the frequency have on the piston’s velocity, acceleration, and jerk? (this is why machinery breaks down if you run it too fast)

7. Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 7$.

   Your answer should be either in terms of $x$ and $y$, or only in terms of $x$. 