Math 112, Part I Review (WS1 – WS 11)

Very Important:
On a test, unless otherwise stated, always show your work (explain your answers)!
If you work from a graph:

- ✓ **clearly & <u>carefully</u> draw and mark any points or lines you use on the graph**
- ✓ **briefly describe the process you used in the work area for that question.**

<u>I. General Concepts</u>: This part of the course is all about derivatives!

Notation: The derivative of a function $y = f(x)$ is denoted by $f'(x)$ or $\frac{dy}{dx}$

On graph*: f '(m)* is the slope of the tangent line to *f(x)* at the point *x = m*

Main Examples:

- **The derivative of distance is the instantaneous speed.**
  That is, we can measure speeds as slopes of tangent lines to the graph of distance.
- **The derivative of TR is MR (approximately).**
  That is, you can think of MR as slopes of tangents to the graph of TR.
- **The derivative of TC (or VC) is MC (approximately).**
  That is, you can think of MC as slopes of tangents to the graph of TC.

<u>II. We have two methods for computing derivatives</u>. You must be able to do both.

1) **the long way to compute *f '(m):***

   a) **compute the slope of the secant line through the graph of *f(x)***

   at $x = m$ & $x = m+h$:

$$\text{slope of secant} = \frac{f(m+h) - f(m)}{h}.$$

   b) **Simplify this expression to cancel the *h* in the denominator**

   c) **let *h* go to 0 to get the slope of the tangent line, *f '(m)*.**

2) **using the derivative rules.**

   This should be your default method — do this unless you're told otherwise!

   <u>Review the 3 rules we learned and have them handy on your sheet of notes.</u>

<u>III. Relationship between graphs of *f(x)* and *f ’(x)*</u>

Given the graph of *f(x)*, you should be able to determine & sketch the general shape of the graph of the derivative *f ’(m)*:

- **If *f(x)* is increasing, then *f ’(x)* is positive**

  (the graph of *f ’(x)* is above the x-axis).

- **If *f(x)* is decreasing, then *f ’(x)* is negative**

  (the graph of *f ’(x)* is below the x-axis).

- **If *f(x)* has a horizontal tangent, then *f ’(x)*= 0**

  (the graph of *f ’(x)* is intersecting the x-axis).

Vice versa, given the derived graph (i.e. the graph of *f ’(x)*), you should be able to determine the general shape of *f(x)*:

- **If *f ’(x)* is positive, then *f(x)* is increasing.**
- **If *f ’(x)* is negative, then *f(x)* is decreasing.**
- **If *f ’(a)* = 0 (that is, the derived graph crosses the x-axis at x=a),**

  then *f(x)* has a horizontal tangent line at x=a.

  This may mean that *f(x)* has a local max (“peak”), a local min (“valley”), or neither at x=a, depending on the sign of the derivative before and after the point x=a.

Rules for Rewriting Exponents (give examples of each):

- $x^a x^b = x^{a+b}$
- $\frac{x^a}{x^b} = x^{a-b}$
- $x^{-a} = \frac{1}{x^a}$; $\frac{1}{x^{-a}} = x^a$
- $x^{a/b} = \sqrt[b]{x^a}$
- $(x^a)^b = x^{ab}$
- $x^0 = 1$
- $x^1 = x$

Add the derivative rules, with examples.