Winter 2008 (Nichifor), Lecture C, version 1

1) (15 points) Evaluate the indicated derivatives of the following functions. **Do not simplify**.

a) $f(x) = \left(1 + e^x + \sqrt{x}\right)(1 + x^3)$

(Product Rule)

$$\boldsymbol{f'(x) = \left(1 + e^x + \sqrt{x}\right)'\left(1 + x^3\right) + \left(1 + e^x + \sqrt{x}\right)\left(1 + x^3\right)'}$$

$$\boldsymbol{= \left(e^x + \frac{1}{2}x^{-\frac{1}{2}}\right)\left(1 + x^3\right) + \left(1 + e^x + \sqrt{x}\right)\left(3x^2\right)}$$

b) $z = \frac{xe^x}{3x+1}$

(Quotient Rule, combined with Product Rule)

$$\boldsymbol{\frac{dz}{dx} = \frac{(e^x + xe^x)(3x + 1) - (xe^x)(3)}{(3x + 1)^2}}$$

c) $g(t) = \sqrt{1 + \ln(3t^2)}$

This is a double Chain Rule.

$$\boldsymbol{g'(t) = \frac{1}{2}\left[1 + \ln\left(3t^2\right)\right]^{-\frac{1}{2}}\left(\frac{1}{3t^2}\right)(6t)}$$

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2. (15 Points) You produce and sell plasma TV's and Computer Monitors.

(a) (3 pts) Suppose you sell each TV for $3000 and each Monitor for $400. Give a formula for the total revenue $R(x, y)$, in dollars, which results from selling x TV's and y Monitors.

ANSWER: $R(x, y)$ =**3000x+400y**

(b) Suppose your profit from selling x TV's and y Monitors is given by the function:

$$P(x, y) = 0.2x^2 + 0.1\, y^2 - 0.3xy + 800x + 200y - 1000$$

i. (4 pts) Compute the two partial derivatives of your Profit function.

$$P_x(x, y) = \mathbf{0.4x - 0.3y + 800}$$

$$P_y(x, y) = \mathbf{0.2y - 0.3x + 200}$$

ii. (5 pts) Find all candidates (x, y) for local minima or maxima of $P(x, y)$.

We need to solve the system: $\begin{cases} 0.4x - 0.3y + 800 = 0 \\ 0.2y - 0.3x + 200 = 0 \end{cases}$

There are a number of correct ways to proceed. Here's one:
Solving for y in the 2nd equation: $y = 1.5x - 1000.$ **Substituting in the first:**

$$0.4x - 0.3(1.5x - 1000) + 800 = 0$$

$$0.05x = 1100$$

$$x = 22{,}000$$

$y = 1.5(22{,}000) - 1000 = 32{,}000$

Note: if you chose to start off by solving for, say, y in the first equation, so you needed to divide by 0.3, you cannot simply round off your coefficients – or else your answer will be off by quite a bit! You need to work with $y = \frac{0.4}{0.3}x + \frac{800}{0.3} = \frac{4}{3}x + \frac{8000}{3}$, *NOT y=1.33x+2666.67).*
Of course, it's best to just solve for something with easier coefficients, like x in the 1st equation, or y in the 2nd.

Answer: (x, y) = **(22,000, 32,000)**

iii. (3 pts) Suppose you've sold 150 TV's and 350 Monitors. Use a partial derivative to estimate the increase in your profit if you sell one more TV.

$$\frac{P(151,350) - P(150,350)}{1} \cong P_x(150,350) = 0.4(150) - 0.3(350) + 800 = 755$$

Answer: Profit will change by about $ **755**

3. (20 points)

The **Marginal Cost**, in dollars, for producing q thousand Items is given by the function:

$$MC(q) = \frac{1}{20}q^3 - 0.6q^2 + 2q + 12$$

a) (5 pts) Find all quantities where the Marginal Cost function has a horizontal tangent.

Round your answer(s) to the nearest two decimal digits.

$$\boldsymbol{MC'(q) = \frac{3}{20}q^2 - 1.2q + 2}$$

Set $MC'(q) = 0$ and use the Quadratic Formula to get the answers

Answer: $MC(q)$ has horizontal tangents at q =**2.37 and 5.63** (*list all*)

b) (6 pts) What is the lowest value of the Marginal Cost on the interval from $q = 0$ to $q = 4$ thousand Items? Show your steps.

We evaluate the MC at any critical point in the interval (q=2.37) and at the endpoints.

- $\boldsymbol{MC(0) = 12}$
- $\boldsymbol{MC(2.37) = \frac{1}{20}(2.37)^3 - 0.6(2.37)^2 + 2(2.37) + 12 = 14.04}$
- $\boldsymbol{MC(4) = \frac{1}{20}(4)^3 - 0.6(4)^2 + 2(4) + 12 = 13.06}$

Answer: Lowest MC is $___**12**____

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Recall that the **Marginal Cost** for producing q thousand Items is given by the function:

$$MC(q) = \frac{1}{20}q^3 - 0.6q^2 + 2q + 12$$

c) (5 pts) Apply the Second Derivative Test to each value you found in part (a). State clearly what the results of your test tell you about the Marginal Cost function at these specific points, and circle your answers.

We've already computed: $MC'(q) = \frac{3}{20}q^2 - 1.2q + 2$. Taking the 2nd derivative:

$$MC''(q) = \frac{6}{20}q - 1.2$$

Evaluating the 2nd derivative of MC at the critical points we found in part (a):

$$MC''(2.37) = \frac{6}{20}(2.37) - 1.2 < 0$$

$$MC''(5.63) = \frac{6}{20}(5.63) - 1.2 > 0$$

According to the 2nd derivative test, MC has a local maximum at q=2.37, and a local minimum at q=5.63.

d) (4 pts) Is the graph of the **Total Cost** concave up or concave down at the point $q = 10$? Justify your answer.

The sign of the 2nd derivative of the Total Cost function tells us the concavity of TC's graph.

We're not given the TC function, but we know the MC function, and we know that TC'= MC.

So: $TC''(q) = MC'(q) = \frac{3}{20}q^2 - 1.2q + 2$

Evaluating at $q = 10$: $TC''(10) = \frac{3}{20}(10)^2 - 1.2(10) + 2 = 5 > 0$

Since TC''(10)>0, the graph of TC is concave up at q=10

Answer: $TC(q)$ is concave _____**UP**______ at $q = 10$.