1. (5 points) **Find $f'(x)$, if $f(x) = x\left(\sqrt{x} + \frac{2}{x^2}\right) - 3x+1$.**

First, rewrite f(x) as a sum of powers of *x*, by distributing the *x* across and using the exponent rules:

$$f(x) = x\sqrt{x} + x\frac{2}{x^2} - 3x + 1 = x \cdot x^{\frac{1}{2}} + 2x \cdot x^{-2} - 3x + 1 = x^{\frac{3}{2}} + 2x^{-1} - 3x + 1$$

Then compute the derivative using the sum, coefficient, and power rules:

$$f'(x) = \left(\frac{3}{2}\right)x^{\frac{1}{2}} - 2x^{-2} - 3 + 0$$

You may rewrite this (though you don't have to) as:

$$f'(x) = \frac{3}{2}\sqrt{x} - \frac{2}{x^2} - 3$$

Answer: $f'(x) = \left(\frac{3}{2}\right)x^{\frac{1}{2}} - 2x^{-2} - 3$

2. (9 points) **Let $A(t)$ represent the amount of water (in thousands of gallons) that flows into a reservoir in *t* hours. We are not told the formula for $A(t)$, but instead are told that:**

$$\frac{A(m+h) - A(m)}{h} = 17 - 3m + h$$

(a) Compute the average rate of flow into the reservoir during the half-hour interval beginning at t = 3.

Compute this directly from the given formula, by substituting m=3, and h=0.5:

$$\frac{A(3+0.5) - A(3)}{0.5} = 17 - 3(3) + 0.5 = 8.5$$

Answer: **8.5** thousand gallons per hour

(b) Find a positive value of *h* so that $A(5+h) - A(5) = 10$.

First, substitute m=5 in the given formula: $\frac{A(5+h)-A(5)}{h} = 17 - 3(5) + h = 2 + h$

Now multiply by h on both sides, so as to cancel the denominator: $h\frac{A(5+h)-A(5)}{h} = h(2+h)$.

We get the formula: $A(5+h) - A(5) = h(2+h)$

Setting this equal to 10, we get: $h(2+h) = 10$, i.e. $h^2 + 2h - 10 = 0$

Applying the Quadratic Formula (or factoring) we get: $h \approx -4.32$ and $h \approx 2.32$

Answer: *h* = **2.32**

(c) Compute the instantaneous rate of flow into the reservoir at t = 3.5 hours. Give correct units in your answer.

We are asked to compute $A'(3.5)$.

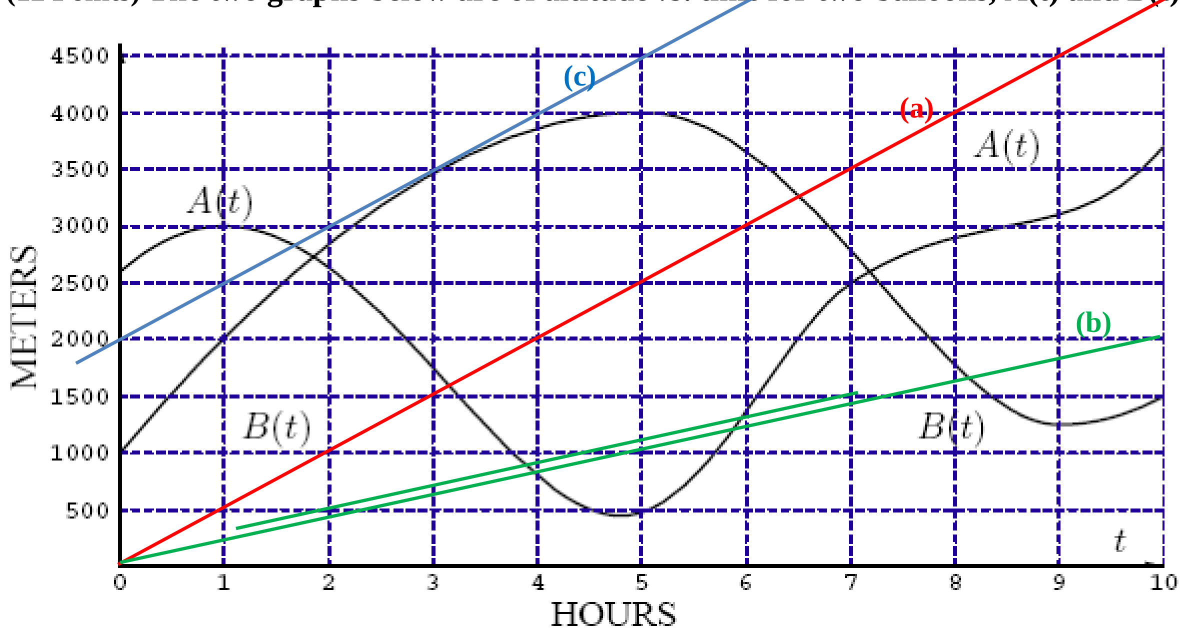
One way to do this is to first find a formula for $A'(t)$, then evaluate it at 3.5:

$\frac{A(t+h)-A(t)}{h} = 17 - 3t + h$. Let h→0, to get $A'(t) = 17 - 3t$. Then, $A'(3.5) = 17 - 3(3.5) = 6.5$

Alternatively, $\frac{A(3.5+h)-A(3.5)}{h} = 17 - 3(3.5) + h = 6.5 + h$. Letting h go to zero, we get $A'(3.5) = 6.5$

Answer: **6.5 thousand gallons per hour**

3. (12 Points) The two graphs below are of altitude *vs*. time for two balloons, A(t) and B(t).



a) **Find a time interval during which balloon B ascends (rises) at a speed greater than 500 meters per hour.**

Draw a reference line of slope 500 (see red line).
Slide it to identify the points where the speed of B equals 500 m/hr: $at\ t \approx 3$.
Roll the ruler along the graph of B: the slopes of the tangent lines are steeper than those of the reference line between 0 and about 3 hrs.

Answer: From t =____**0**_____ to t =_____**3**______ hours.

b) **Find a time *t* such that** $\frac{A(t+2)-A(t)}{2} = 200$.

We want a time t such that the slope of the secant line through the graph of A(t) at t and t+2 is 200.
Draw a reference line of slope 200 (see green line). Slide the ruler parallel to it until it crosses the graph of A(t) at two points which are two hours apart. (the reference line is almost the line you want, but not quite.)

Answer: At t =______**3.9**________ hours.

c) **Estimate the speed of balloon B at t = 3 hours.**

Draw a tangent line to the graph of B(t) at t=3hrs.

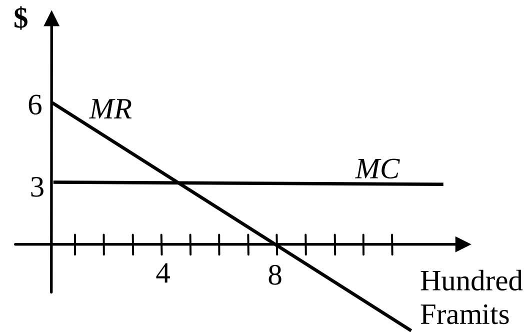
2 points on this line are, for example, (0,2000) and (3, 3500).

Slope=1500/3=500.

Answer: _____**500**_______ meters per hours.

4. (11 points = 3+3+5)

You produce and sell Framits. The graphs to the right show your **Marginal Revenue and Marginal Cost** at q hundred Framits. You need not show work in this problem.



a) What quantity between q=1 and q=3 hundred Framits results in the largest Profit?

Answer: q=____3_____hundred Framits.

(as long as MR>MC, the profit keeps increasing!)

b) What quantity between q=1 and q=3 hundred Framits results in the largest Total Revenue?

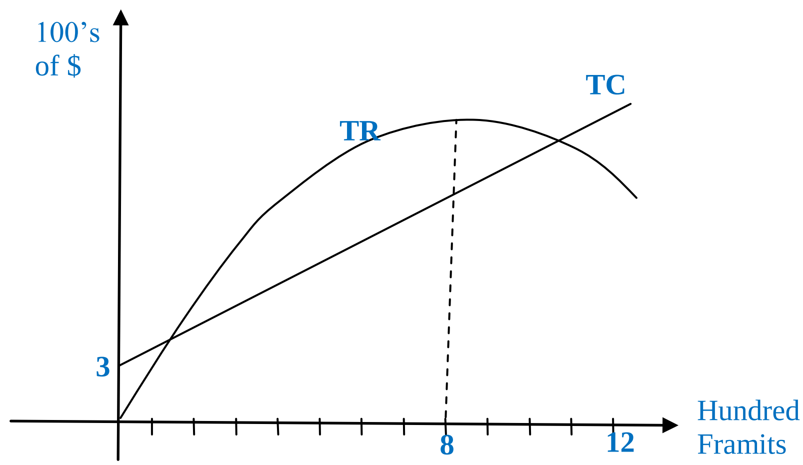
Answer: q=______3______ hundred Framits.

(recall that MR≈TR'. So, as long as MR is positive, TR keeps increasing)

c) Your Fixed Costs are FC = $300. You may assume that TR(0)=$0.

On the axes below, sketch the graphs of the Total Revenue and Total Cost.

Label the y-intercepts, and the x-coordinates of the highest or lowest point on the graphs.



Note: THERE WAS NO NEED TO COMPUTE FORMULAS FOR TR AND TC, this problem could be done quickly and correctly by just going from the provided MR&MC graphs (derived graphs of TR &TC) to the TR&TC graphs.

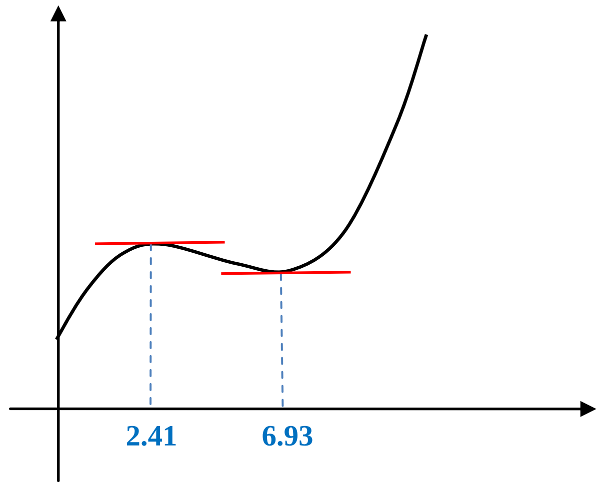
1) *The units should be: q in hundreds of Framits, TR&TC in hundreds of dollars.*
2) *Since MC(q)=3, the Total Cost is a line of slope 3. TC(0)=FC=3 hundred dollars.*
   So, for TC, we need to draw a straight line with y-intercept 3 and slope 3.
3) *The TR graph should start at the origin and increase as long as MR is positive (up to q=8), then decrease.*

We gave credit for: TC drawn as a line of positive slope and y-intercept 3 or 300 (incorrect, but accepted). TR drawn as an increasing then decreasing curve with highest point at q=8 and starting at the origin.

5. (13 Points) The altitude $A(t)$ (in yards) versus time (in hours) for a weather balloon is given by the formula:

$$A(t) = t^3 - 14t^2 + 50t + 30.$$

A sketch of its graph is shown to the right.



a) Write down the formula for the derivative A'(t)

Answer: $A'(t) = 3t^2 - 28t + 50$

b) Find the longest time interval over which the balloon is descending.

Looking at the graph: the balloon is descending between the two points with horizontal tangent lines.

Set $A'(t) = 0$,
and solve the resulting quadratic equation :

$$3t^2 - 28t + 50 = 0$$

The quadratic formula gives solutions: $t \approx 2.41$ and $t \approx 6.93$

Answer: The balloon is descending from t=__**2.41**____ to t=____**6.93**____ hours.

c) Find the lowest altitude the balloon reaches in the time interval t=2 to t=8 hours.

Looking at the graph of the altitude over the listed interval from 2 to 8 hours, and at the two points we obtained in part (b), we see that the balloon reaches its lowest altitude at $t \approx 6.93$ hrs.
Evaluating the altitude at that time:

$$A(6.93) = (6.93)^3 - 13(6.93)^2 + 50(6.93) + 50 \approx 104.99$$

Answer: ____**104.99**_____ yards.