1. (5 points) **Find $f'(x)$, if $f(x) = x\left(\sqrt{x} + \frac{7}{x^2}\right) - 8x + 1$.**

First, rewrite f(x) as a sum of powers of *x*, by distributing the *x* across and using the exponent rules:

$$f(x) = x\sqrt{x} + x\frac{7}{x^2} - 8x + 1 = x \cdot x^{\frac{1}{2}} + 7x \cdot x^{-2} - 8x + 1 = x^{\frac{3}{2}} + 7x^{-1} - 8x + 1$$

Then compute the derivative using the sum, coefficient, and power rules:

$$f'(x) = \left(\frac{3}{2}\right)x^{\frac{1}{2}} - 7x^{-2} - 8 + 0$$

You may rewrite this (though you don't have to) as:

$$f'(x) = \frac{3}{2}\sqrt{x} - \frac{7}{x^2} - 8$$

Answer: $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 7x^{-2} - 8$

2. (9 points) **Let $A(t)$ represent the amount of water (in thousands of gallons) that flows into a reservoir in *t* hours. We are not told the formula for $A(t)$, but instead are told that:**

$$\frac{A(m+h) - A(m)}{h} = 15 - 3m + h$$

(a) Compute the average rate of flow into the reservoir during the half-hour interval beginning at t = 2.

Compute this directly from the given formula, by substituting m=2, and h=0.5:

$$\frac{A(2+0.5) - A(2)}{0.5} = 15 - 3(2) + 0.5 = 9.5$$

Answer: **9.5** thousand gallons per hour

(b) Find a positive value of *h* so that $A(6+h) - A(6) = 10$.

First, substitute m=6 in the given formula: $\frac{A(6+h)-A(6)}{h} = 15 - 3(6) + h = h - 3$

Now multiply by h on both sides, so as to cancel the denominator: $h\frac{A(6+h)-A(6)}{h} = h(h-3)$.

We get the formula: $A(6+h) - A(6) = h(h-3)$

Setting this equal to 10, we get: $h(h-3) = 10$, i.e. $h^2 - 3h - 10 = 0$

Applying the Quadratic Formula (or factoring) we get: $h = -2$ and $h = 5$

Answer: *h* = **5**

(c) Compute the instantaneous rate of flow into the reservoir at t = 3.12 hrs. Give correct units in your answer.

We are asked to compute $A'(3.12)$.

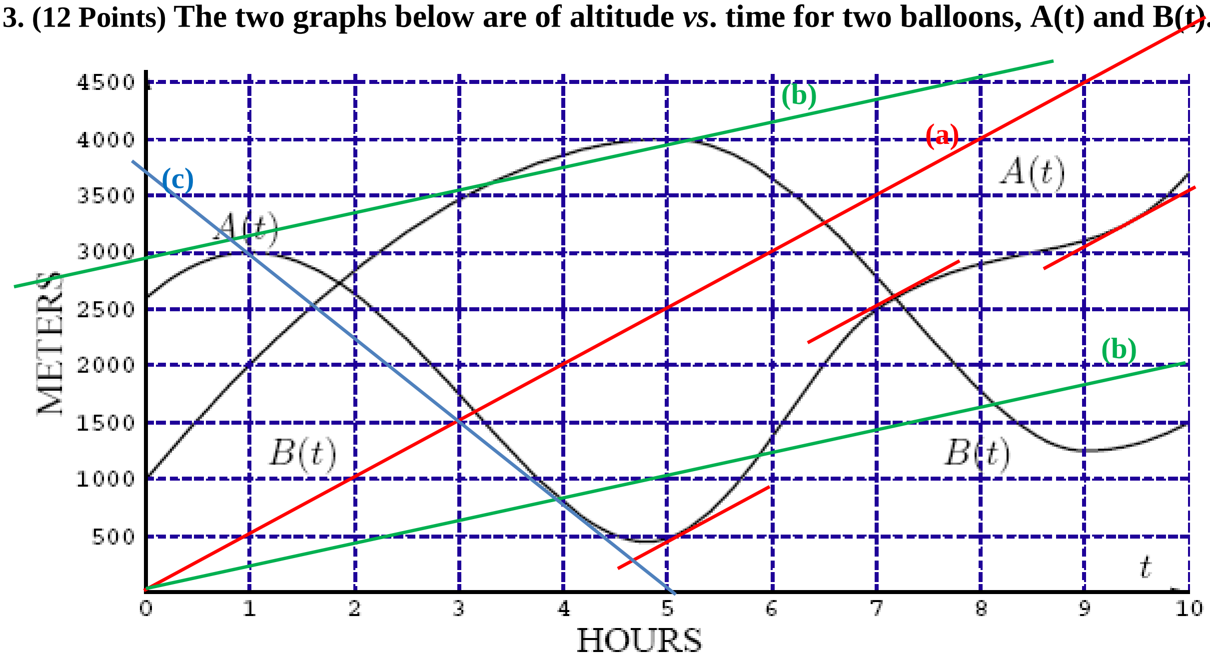
One way to do this is to first find a formula for $A'(t)$, then evaluate it at 3.12:

$\frac{A(t+h)-A(t)}{h} = 15 - 3t + h$. Let h→0, to get $A'(t) = 15 - 3t$. Then, $A'(3.12) = 15 - 3(3.12) = 5.64$.

Alternatively, $\frac{A(3.12+h)-A(3.12)}{h} = 15 - 3(3.12) + h = 5.64 + h$. Letting h go to zero, we get $A'(3.12) = 5.64$

Answer: **5.64 thousand gallons per hour**

3. (12 Points) The two graphs below are of altitude *vs*. time for two balloons, A(t) and B(t).



a) Find a time interval during which balloon A ascends (rises) at a speed greater than 500 meters per hour.

Draw a reference line of slope 500 (see red line).
Slide it to identify the points where the speed of A equals 500 m/hr: $at\ t \approx 5.1, t \approx 7.2, and\ t \approx 9.5\ hrs$
Roll the ruler along the graph of A:the slopes of the tangent lines are steeper than those of the reference line between 5.1 and 7.3, or after 9.5.

Answer: From t =___5.2___ to t =____7.1______ hours.

b) Find a time *t* such that $\frac{B(t+2)-B(t)}{2} = 200$.

We want a time t such that the slope of the secant line through the graph of B(t) at t and t+2 is 200.
Draw a reference line of slope 200 (see green line). Slide the ruler parallel to it until it crosses the graph of B(t) at two points which are two hours apart.

Answer: At t =__3.2___ hours.

c) Estimate the speed of balloon A at t = 4 hours.

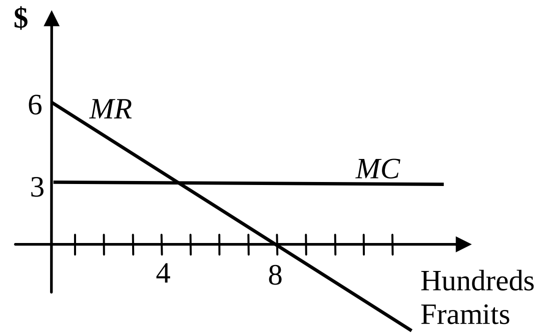
Draw a tangent like to the graph of A(t) at t=4hrs. Pick 2 points on this line, say (1, 3000) & (3, 1500), and compute its slope: $\frac{1500-3000}{3-1} = -750$

Answer: ____-750_____ meters per hours.

4. (11 points = 3+3+5)

You produce and sell Framits. The graphs to the right show your **Marginal Revenue and Marginal Cost** at q hundred Framits.

You need not show work in this problem.



a) What quantity between q=2 and q=4 hundred Framits results in the largest profit?

Answer: q=____4______hundred Framits.

(as long as MR>MC, the profit keeps increasing!)

b) What quantity between q=2 and q=4 hundred Framits results in the largest Total Revenue?

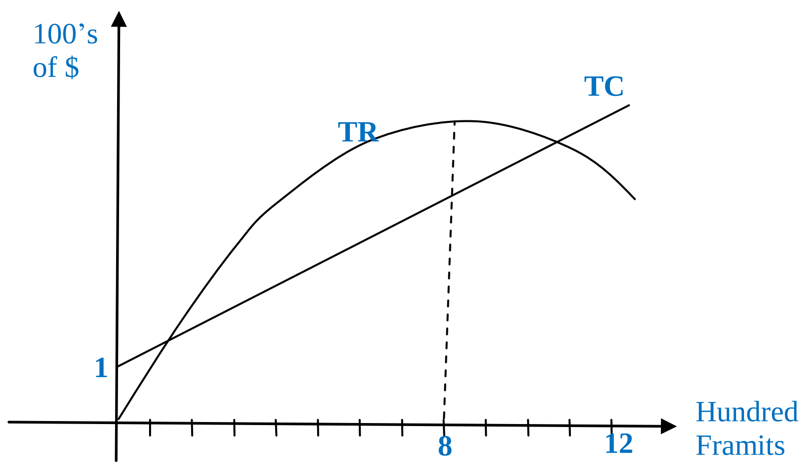
Answer: q=_____4______ hundred Framits.

(recall that MR≈TR'. So, as long as MR is positive, TR keeps increasing)

c) Your Fixed Costs are FC = $100. You may assume that TR(0)=$0.

On the axes below, sketch the graphs of the Total Revenue and Total Cost.

Label the y-intercepts, and the x-coordinates of the highest or lowest point on the graphs.



Note: THERE WAS NO NEED TO COMPUTE FORMULAS FOR TR AND TC, this problem could be done quickly and correctly by just going from the provided MR&MC graphs (derived graphs of TR &TC) to the TR&TC graphs.

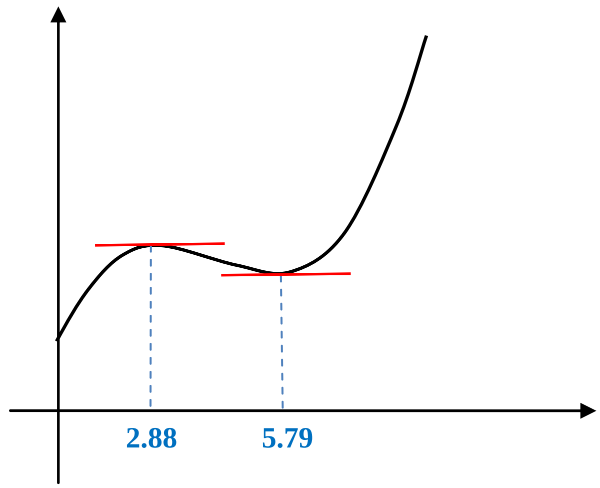
1) *The units should be: q in hundreds of Framits, TR&TC in hundreds of dollars.*
2) *Since MC(q)=3, the Total Cost is a line of slope 3. TC(0)=FC=1 hundred dollars.*
   So, for TC, we need to draw a straight line with y-intercept 1 and slope 3.
3) *The TR graph should start at the origin and increase as long as MR is positive (up to q=8), then decrease.*

We gave credit for: TC drawn as a line of positive slope and y-intercept 1 or 100 (incorrect, but accepted). TR drawn as an increasing then decreasing curve with highest point at q=8 and starting at the origin.

5. (13 Points) The altitude $A(t)$ (in yards) versus time (in hours) for a weather balloon is given by the formula:

$$A(t) = t^3 - 13t^2 + 50t + 50.$$

A sketch of its graph is shown to the right.



a) Write down the formula for the derivative A'(t)

Answer: $A'(t) = 3t^2 - 26t + 50$

b) Find the longest time interval over which the balloon is descending.

Looking at the graph: the balloon is descending between the two points with horizontal tangent lines.

Set $A'(t) = 0$,
and solve the resulting quadratic equation :

$$3t^2 - 26t + 50 = 0$$

The quadratic formula gives solutions: $t \approx 2.88$ and $t \approx 5.79$

Answer: The balloon is descending from t=__2.88____ to t=_____5.79_____ hours.

c) Find the lowest altitude the balloon reaches in the time interval $t = 2$ to $t = 8$ hours.

Looking at the graph of the altitude over the listed interval from 2 to 8 hours, and at the two points we obtained in part (b), we see that the balloon reaches its lowest altitude at $t \approx 5.79$ hrs.
Evaluating the altitude at that time:

$$A(5.79) = (5.79)^3 - 13(5.79)^2 + 50(5.79) + 50 \approx 97.79$$

Answer: _______97.79______ yards.