1) (12 points) Show your work.

a) Find *f'(x)*, if $f(x) = 5(1-2x)^2$.

First, rewrite f(x) as a sum, by squaring the expression and distributing the coefficient across:

$$f(x) = 5(1-4x+4x^2) = 5 - 20x + 20x^2$$

Then differentiate using the sum, coefficient, and power rules:

$$f'(x) = 0 - 20 \cdot 1 + 20 \cdot (2x)$$

Answer: *f'(x)* = ____40x-20______

b) Find $\frac{dy}{dx}$, if $y = 3\sqrt[5]{x} + \frac{2}{x^3} - 3$. You do not need to simplify your answer.

First, rewrite y as a sum of powers of x, using the exponent rules:

$$y = 3x^{1/5} + 2x^{-3} - 3$$

Then differentiate using sum, coefficient, and power rules:

$$y' = 3\left(\frac{1}{5}\right)x^{\frac{1}{5}-1} + 2(-3)x^{-3-1} - 0 = \frac{3}{5}x^{-\frac{4}{5}} - 6x^{-4}$$

Answer: $\frac{dy}{dx} = \frac{3}{5}x^{-4/5} - 6x^{-4}$

c) The derivative of a certain unknown function $g(x)$ is $g'(x) = \frac{x^2}{x+1}$. Use this fact to approximate the value of

$$\frac{g(5.002)-g(5)}{0.002}$$

Recall that the main idea in the first few worksheets was that the slope of the tangent line at a point (a.k.a. the derivative) can be approximated by the slope of the secant line through that point and a nearby one.

The expression you're given is the slope of the secant line through the graph of g(x) at x=5 and x=5.002.

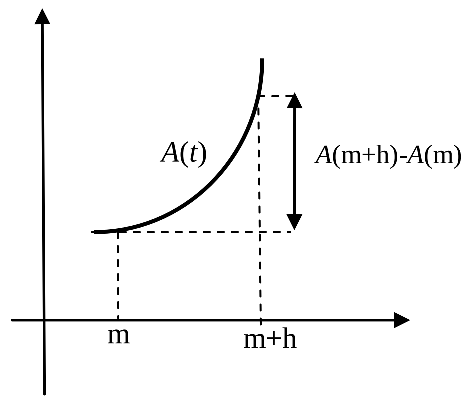
Hence,

$$\frac{g(5.002)-g(5)}{0.002} \approx g'(5) = \frac{5^2}{5+1} = \frac{25}{6}$$

Answer: $\frac{g(5.002)-g(5)}{0.002} \approx \frac{25}{6}, or\ 4.17$

2. (10 points)

To the right is a rough sketch of part of the graph of a function $A(t)$. We are not given a formula for $A(t)$, but we are told instead that:



$$A(m+h) - A(m) = \frac{2mh}{(m+1)(m+h)}$$

a) Compute the slope of the secant line to the graph of A(t) from t = 1 to t = 5.

We are asked to compute $\frac{A(5)-A(1)}{4}$.
We can plug in m=1 and h=4 in the provided formula to get the ***top*** *part of this expression (the rise), then we need to divide by 4 (the run) to get the slope:*

$$A(5) - A(1) = \frac{2(1)(4)}{(1+1)(1+4)} = \frac{8}{10} = 0.8$$

$$\frac{A(5) - A(1)}{4} = \frac{0.8}{4} = 0.2$$

Answer:________0.2_________

b) Find a formula in terms of t for $A'(t)$. Show all your steps.

1. Slope of secant from t to t+h is:

$$\frac{A(t+h) - A(t)}{h} = \frac{\frac{2th}{(t+1)(t+h)}}{h}$$

2. Simplify (cancel the h in the denominator):

$$\frac{A(t+h) - A(t)}{h} = \frac{1}{h} * \frac{2th}{(t+1)(t+h)} = \frac{2t}{(t+1)(t+h)}$$

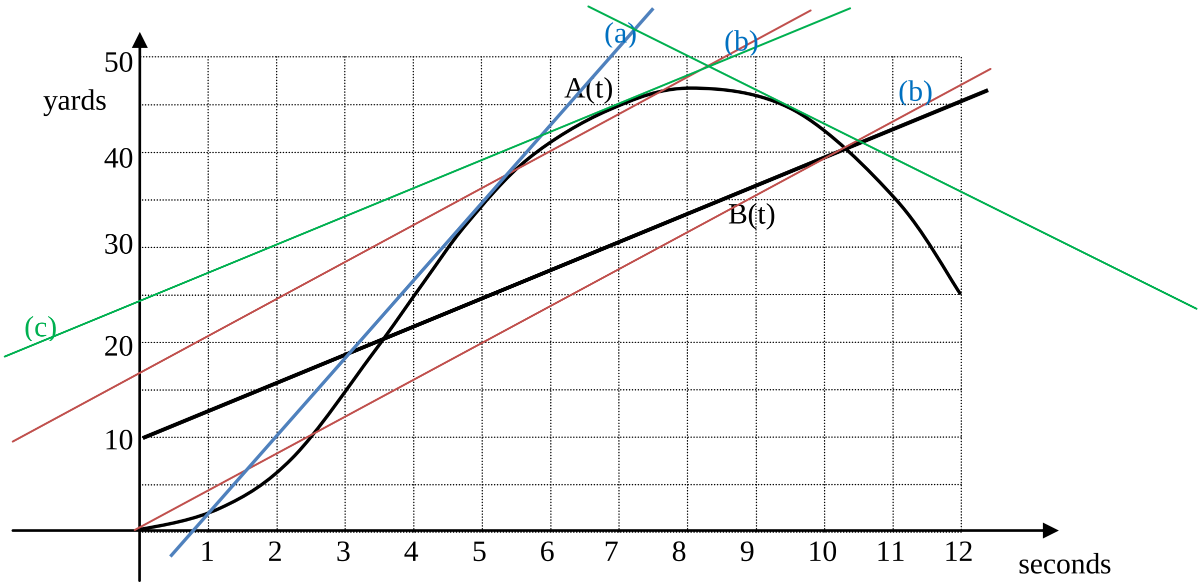
3. Let h → 0, so that slope of secant → slope of tangent =A'(t)

$$A'(t) = \frac{2t}{(t+1)t}$$

Answer can be written as $\frac{2t}{(t+1)t}, \frac{2t}{t^2+t}, or\ \frac{2}{t+1}\ (after\ cancelling\ t)$

Answer: $A'(t) = \frac{2t}{(t+1)t}$

3. (15 Points) The two graphs below are of **distance** *vs*. time for two cars, car A and car B.



a) What is the speed of car A at t=5 seconds?

Draw the tangent line to the A(t) graph at t=5. Compute its slope. Answer should be close to 8.3

Answer: _____8.3___ yards/second

b) Find a 2 second time interval during which the **average** speed of car A is 4 yards/second.
Draw a reference line of slope 4. Slide ruler parallel to this line until it crosses the graph of A at two points which are 2 seconds apart.

Answer: From t=__5.7____ to t=____7.7_____ seconds.

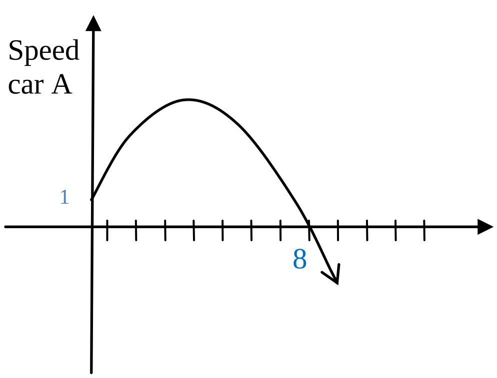
c) Find the longest time interval during which car A is ahead of car B, but car A drives slower than car B.

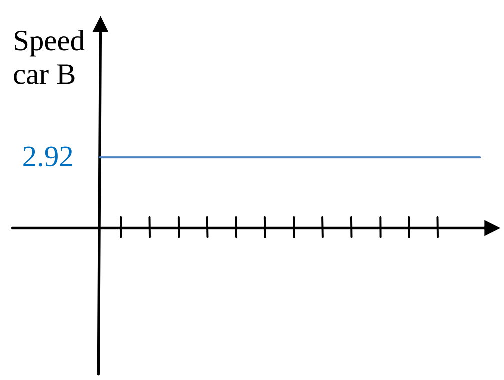
Car A is ahead when the A(t) graph is above: from t=3.7 to t=10.3.
During this interval, the two cars have matching speeds at about t=7 (see green line).

Answer: From t=__7__ to t=__10.3____ seconds.

(also OK: from t=7 to t=9.4, if you exclude the times when the speed of A is negative but faster than that of B)

d) Sketch the **speed** graphs of the two cars. Compute and label the y and x-intercepts, and sketch the general shape of each of the two graphs.

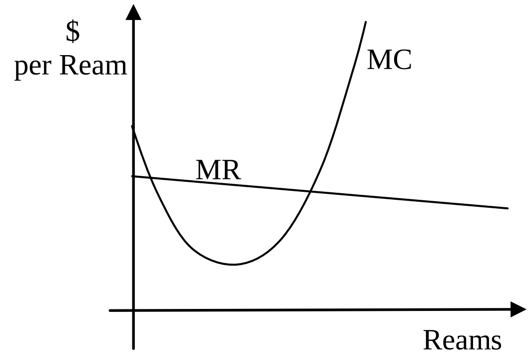




(for car B, speed at any time=slope of B(t) graph≈35/12≈2.92)

(for car A, we accepted y-intercepts between 0 and about 1.25)

4. (13 Points) A print shop produces and sells Reams of paper. The Marginal Cost and Marginal Revenue (in dollars per Ream) for producing q Reams are given by:



$$MR(q) = 5 - 0.04q$$
$$MC(q) = 0.2q^2 - 1.2q + 6$$

a) What quantity results in the greatest profit?
Round your answer to the nearest whole number of Reams.

Set MR=MC and solve for q:

$$5 - 0.04q = 0.2q^2 - 1.2q + 6$$
$$0.2q^2 - 1.16q + 1 = 0$$

Quadratic Formula: q ≈ 1.05, q ≈ 4.75.

Looking at the graphs, MR>MC changes to MR<MC at the second of the two crossing points: q=4.75 Reams.

Answer: q=______**5**__________Reams

b) What is the lowest value of the marginal cost?

MC is a quadratic whose graph is a concave-up parabola, so its lowest value is at its vertex:

$$q = \frac{-b}{2a} = \frac{1.2}{0.4} = 3$$

We are asked for lowest value of MC, so evaluate MC at q=3:

$$MC(3) = 0.2(3)^2 - 1.2(3) + 6 = 4.2$$

Answer: Lowest value of MC is ________**4.2**______dollars

c) The print shop starts selling its paper by the Page instead of by the Ream. One Ream consists of 200 Pages. What is the revenue from selling the 201^{st} Page? (in this question, pay attention to all your units!)

First, note that we are really asked to find the marginal revenue (in dollars per page!) at 200 pages =1 Ream.
Since q is measured in Reams, we need to compute MR(1)=5-0.04=4.96.
However, the units for the MR function are dollars per ream.
So we also need to convert 4.96 dollars per ream into dollars per page: 4.96/200=0.0248.

Answer: ______**0.0248**_______ dollars per Page