

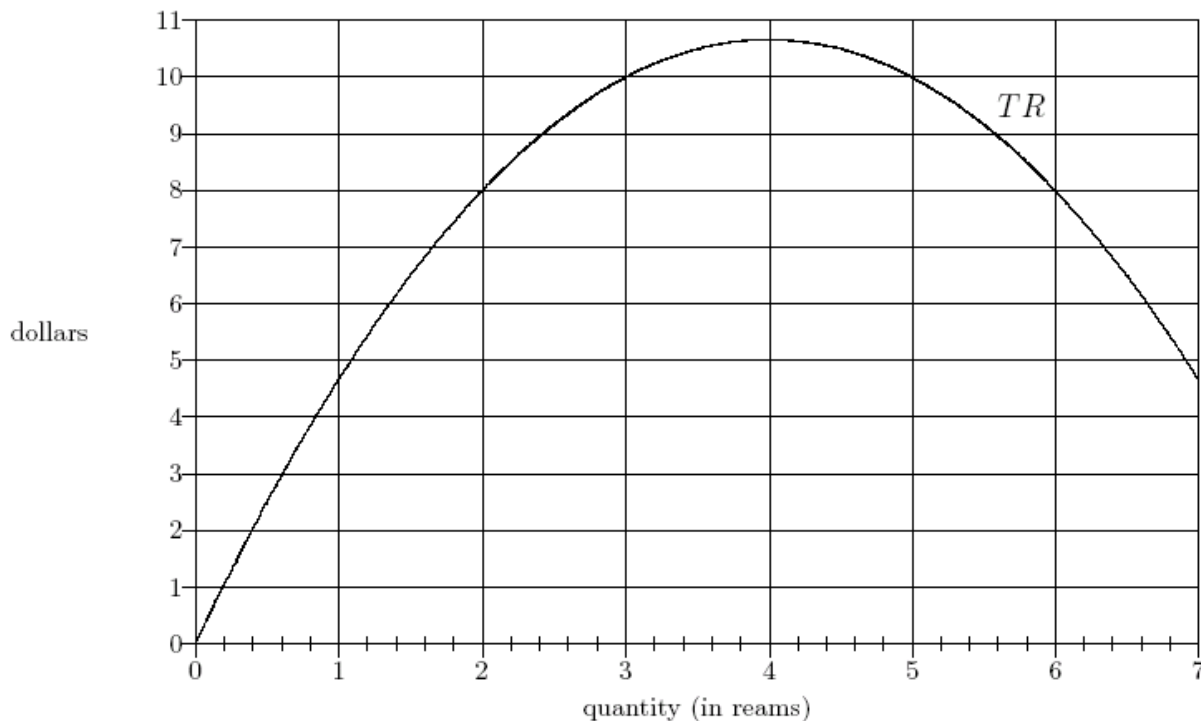
## WS 2: Two versions of MR and MC

### **Main Idea:**

If we're given the graph of TR (or TC),  
and if the units are large  
(that is, a change of 1 unit on the x-axis is not legible)  
then the slope of the tangent line approximates MR (or MC)  
-- but we have to make sure we compute it in the correct units!!

### **Example:**

**You own a print shop. The graph in the handout (from the package of lecture handouts on the class website) shows the graph of your total revenue TR.**



From the graph:

An order of 1 ream results in a total revenue of:  $TR(1) = \$4.75$

An order of 2 reams results in a total revenue of:  $TR(2) = \$8$

An order of 3 reams results in a total revenue of:  $TR(3) = \$10$

Q1: What is the marginal revenue at 2 reams?

1. Recall the definition of MR:

The marginal revenue at quantity  $q$  is the change in TR as we move from  $q$  to  $q+1$  (sell one more unit).

$$\mathbf{MR(q)=TR(q+1)-TR(q)}$$

Here, we're working with  $q$  in reams, so **1 unit=1 Ream**

A1:  $MR(2)=TR(3)-TR(2)=$10-$8=$2.$

2. Recall the other way to think of MR:

**MR(q)= slope of secant line thru the TR graph at  $q$  and  $q+1$**

A1:

$$MR(2) = \frac{TR(3) - TR(2)}{3 - 2} = \frac{\$10 - \$8}{1 \text{ ream}} = \$2 / \text{ream}$$

NOTE: Both versions of MR (as increment and as slope) give the same numerical value, but the units are different:

- As increment, MR is in \$
- As slope, MR is in \$ **per unit**

## 2<sup>nd</sup> Scenario:

Suppose you start selling paper in units of 100 pages

That is: **1 unit=1 hundred pages (HP)**

(and 1 ream =500 pages =5HP, 1HP =1/5 reams=0.2 reams)

Q2: Compute MR at 10 HP (=2 reams), both ways.

1. As Increment (recall: 1 unit=1HP):

$$\text{MR}(10 \text{ HP}) = \text{TR}(11 \text{ HP}) - \text{TR}(10 \text{ HP})$$

$$= \text{TR}(2.2 \text{ reams}) - \text{TR}(2 \text{ reams})$$

$$= \$0.5$$

2. As slope:

- Draw a secant through TR at 2 reams and 2.2 reams.
- Find two good points on this line: (0, 3) and (2, 8)
- $\text{MR} = \text{slope} = 5/2 = 2.5$

Huh? How come it does not match the value 0.5 above?!

What are the units? 2.50 is in “\$ per Ream”.

Aha! Should be \$ per unit, i.e. \$ per HP!

So the problem is that we're not done -- we need another step:

- convert \$2.50 per ream to \$ ?? per HP

$$\$2.50 / \text{Ream} = \$2.50 / 5 \text{ HP} = \$0.5 \text{ per HP.}$$

### 3<sup>rd</sup> Scenario:

Suppose you now start selling by the page.

That is: **1 unit=1 page**

(and 1 ream =500 pages, 1 page = 1/500=0.002 reams, etc)

Q3: Compute MR at 500 pages (=1 ream).

1. As Increment (with 1 unit=1page):

$$\text{MR}(500) = \text{TR}(501) - \text{TR}(500)$$

$$= \text{TR}(1.002 \text{ reams}) - \text{TR}(1 \text{ reams})$$

Impossible to read! Points differing by 1 page are too close!

2. We have to compute it as a slope:

- Do your best to draw a secant through TR at 1 Reams and at 1.002 Reams. (It should look like the tangent at 1 ream)
- Find two good points on this line:

$$(0, 0.9) \text{ and } (2.6, 11)$$

- $\text{MR} = \text{slope} = (11 - 0.9) / (2.6 - 0) = \$3.88 \text{ per Ream}$
- Now convert to the correct units:  
 $\$3.88 / \text{ream} = \$3.88 / 500 \text{ pages} = \$0.00776 / \text{page}$

A3: When 1 unit = 1 page,  $\text{MR}(500) = 0.00776$  (\$ per page)

Finally, suppose you have no fixed costs ( $FC=\$0$ ), but it costs you \$2 to produce every 3 Reams.

Q: What quantity maximizes your profit?

Recall: Profit is maximal when we have:

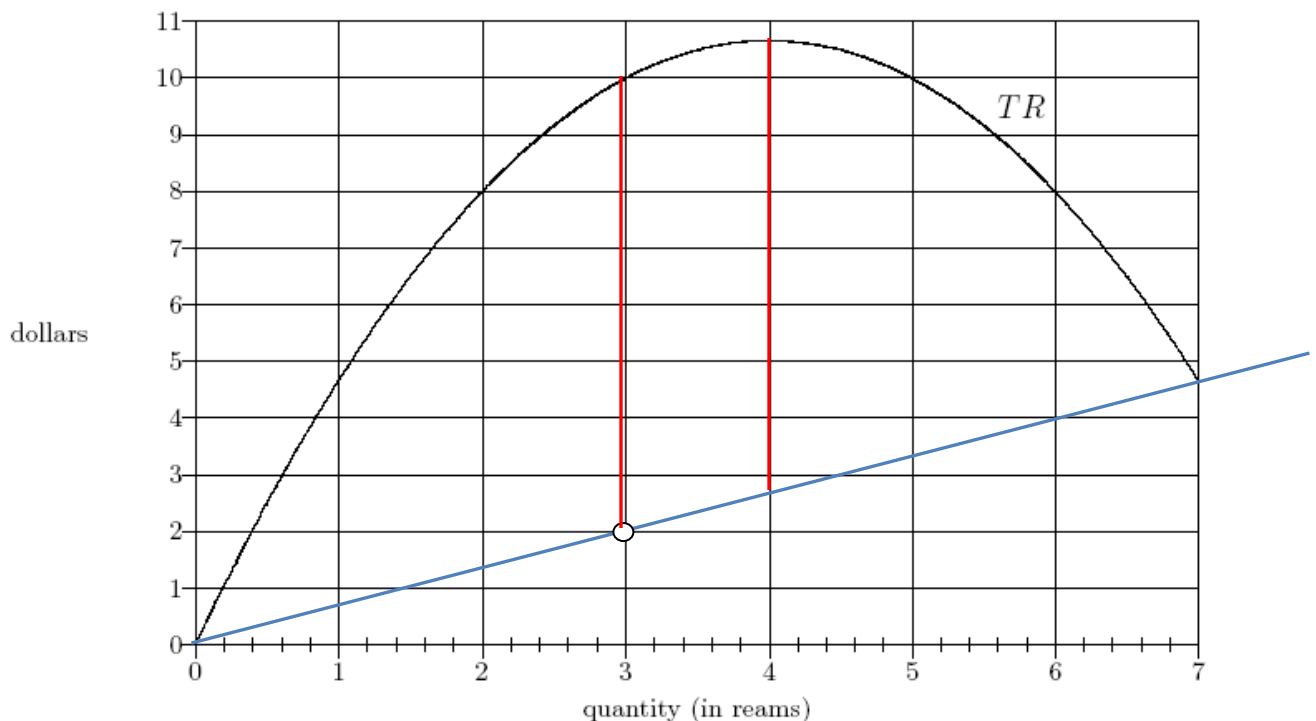
- 1) largest difference (vertical gap) between TR and TC  
(with TR above TC!)
- 2)  $MR=MC$  (matching slopes)  
(transition from  $MR>MC$  to  $MR<MC$ )

Since our costs are \$2 for each 3Reams, every extra Ream costs \$2/3 (per Ream), so the Marginal Cost is constant:

$$MC(q)=2/3 \text{ \$ per Ream.}$$

Further,  $TC(q)=2/3q+0=2/3q$ , with  $q$  measured in Reams.

So, **TC is a diagonal line of slope 2/3.**



Now it's time to notice that the question is not explicit enough. Are we assuming that the quantity  $q$  must be a whole number of reams, or can we sell in HP's or pages?

1) If  $q$  must be a whole number of reams, it's enough to compare the vertical distance at  $q=1, 2, 3, 4, 5$ , etc Reams. We see that the max "gap" is at  $q=3$  or  $q=4$  Reams. So: if we sell by the ream, our **max profit is about \$8**, when we sell  $q=3$  or  $q=4$  Reams.

2) If we sell by the sheet of paper, then we'd have to compare all intermediate points too, which is tough to do well "by eye" alone.

Here, it's best to use the second method: find  $MR=MC$ , i.e. find a quantity where the slopes match.

Since  $TC$  is already a line, we want a point on the graph of Total Revenue  $TR$  where the tangent is parallel to  $TC$ .

Use Rolling Ruler: at about  $q=3.5$  Reams, or 1750 pages.

**Max profit = \$8.2**, at  $q=1750$  pages

