NAME: _____ Student ID #: _____

QUIZ SECTION:_____

Math 112 A
Midterm I
April 24, 2007

Problem 1	10	
Problem 2	10	
Problem 3	15	
Problem 4	15	
Total:	**50**	

- You are allowed to use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 5 pages in total and 4 problems. Make sure you have a complete test.
- Unless otherwise instructed, you **must show how you get your answers**.
  Correct (or incorrect) answers with no supporting work may result in little or no credit.
- If an algebraic method is available, answers obtained by guessing, approximating, or plug-and-check will get little or no credit.
- If you need more room, use the backs of pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

GOOD LUCK

Do you want me to post your grade so far on the class website under the last 4 digits of your STUDENT ID (in about a week)?

☐ *Yes, please post my grade. Sign to give permission:* _____

☐ *No, please don't post my grade so far.*

1) (10 points)

a) Find the derivative $f'(y)$, if $f(y) = \dfrac{5 + y^2 + 3y^3}{y^2}$

$$f(y) = \frac{5}{y^2} + \frac{y^2}{y^2} + \frac{3y^3}{y^2} = 5y^{-2} + 1 + 3y$$

$$f'(y) = -10y^{-3} + 0 + 3$$

Answer: $f'(y) =$ ____$3 - 10y^{-3}$_____

b) Find the derivative $\dfrac{dy}{dx}$, if $y = \sqrt{x^3} - \dfrac{2}{\sqrt[5]{x}} + 7.3$

$$y = x^{\frac{3}{2}} - 2x^{-\frac{1}{5}} + 7.3$$

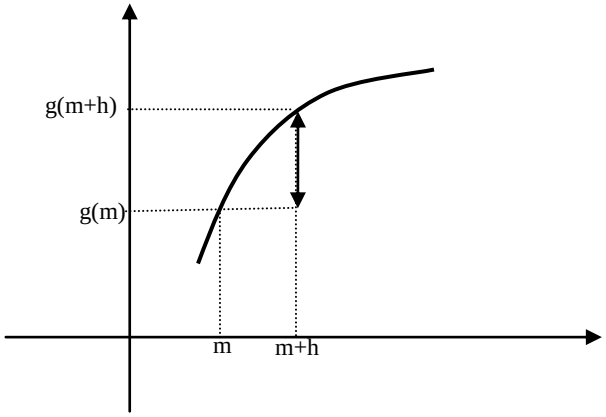
$$y' = \frac{3}{2}x^{\frac{1}{2}} - 2\left(-\frac{1}{5}\right)x^{-\frac{6}{5}} + 0$$

Answer: $\dfrac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + \frac{2}{5}x^{-\frac{6}{5}}$

2. (10 points)

You do not know the formula for the function g*(x)*, but you are told that the rise in the graph of *g(x)* from *x=m* to *x=m+h* is given by the following formula:

$$g(m+h)-g(m)=\frac{mh}{(m+1)(m+h-1)}$$



a) Find the slope of the secant line through the graph of *g(x)* at x=3 and x=5.

$$\frac{g(5)-g(3)}{2}=\frac{1}{2}\frac{(3)(2)}{(3+1)(3+2-1)}=\frac{3}{16}$$

Answer: 3/16

b) Find the slope of the tangent line at x=3.

$$\frac{g(3+h)-g(3)}{h}=\frac{1}{\cancel{h}}\frac{3\cancel{h}}{(3+1)(3+h-1)}=\frac{3}{4(2+h)}$$

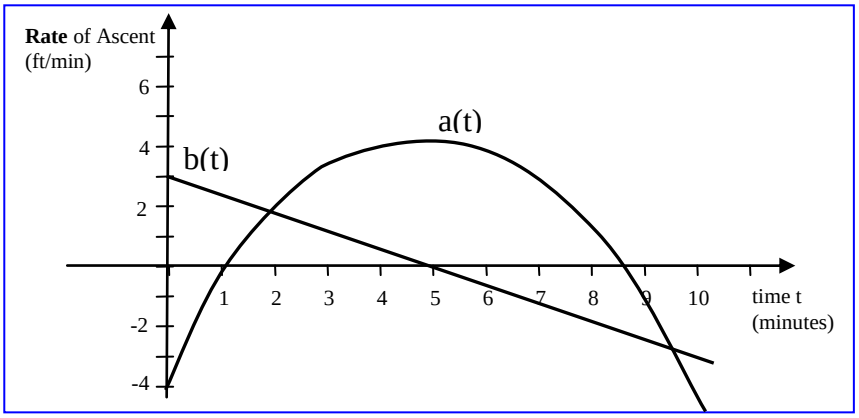
As h→ 0, g'(3)=3/8

Answer: 3/8

3. (15 Points) Two weather balloons, A and B, start off at the same altitude of 100 feet at t=0 and move straight up or down for 10 minutes.

The two graphs at right, labeled a(t) and b(t), represent the respective **rates of ascent** (in feet/minute) for the two balloons. That is, these are the **derived graphs** for the altitudes of the two balloons.

Use these graphs to answer the following questions. No need to show work or explain your answers.



a) How fast is balloon A moving at time t=0, and in what direction?

Answer: At t=0, balloon A is moving up / down at ____4_____ ft/min.

b) Circle the correct answer: At t=5 minutes, balloon **B's altitude** is :

i. Zero feet (that is, balloon B is at ground level)

ii. At its highest above ground

iii. At its lowest above ground

iv. None of the above

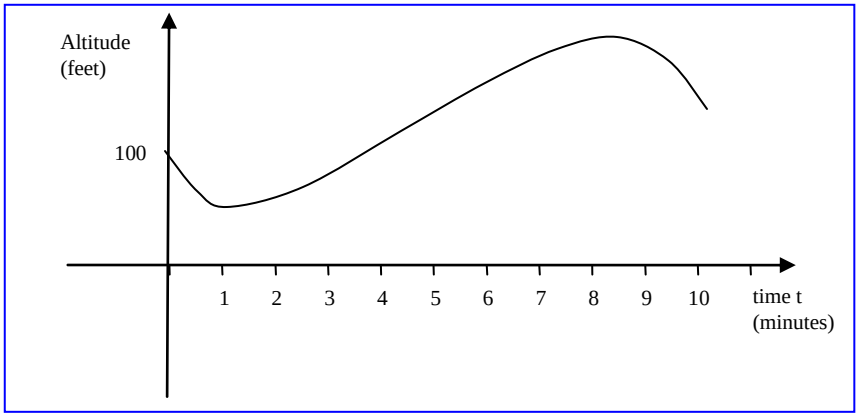
c) Find the longest time interval over which both balloons are rising.

From t =____1_____ to t =____5_____ minutes.

d) In the first 5 minutes, at what time will the vertical distance between the two balloons be greatest? Which balloon is higher above ground at that time, A or B?

At t=____2____ minutes. Balloon __B____ is higher at that time.

e) Sketch the graph of the **altitude** above ground for **balloon A**. Label the y-intercept.



4. (15 Points) Your company, "Stick It", produces and sells Bumper Stickers. The Total Revenue and Total Cost, in **hundreds of dollars**, for producing q **hundred Stickers** are given by the following formulas:

$$TR:\ R(q) = -0.3q^2 + 4q$$

$$TC:\ C(q) = \frac{1}{15}q^3 - \frac{1}{10}q^2 + q + 1$$

a) Find formulas for the Marginal Revenue and the Marginal Cost for producing q hundred Stickers.

Answer: *MR(q)*=____$-0.6q + 4$_____

MC(q)=____$\frac{1}{5}q^2 - \frac{1}{5}q + 1$ _____

*Units for both MR and MC:___$ or $ per Sticker*_____

b) Find the quantity at which the Marginal Cost is lowest.

Since MC is a quadratic whose graph is a concave-up parabola, we can use the vertex formula :

$$q = -\frac{-1/5}{2\left(\frac{1}{5}\right)} = \frac{1}{2} = 0.5$$

Answer: MC is lowest at _____50_____Stickers.

c) What is your maximum profit? (include correct units).

We need to use MR=MC (i.e P'=0):

$-0.6q + 4 = 0.2q^2 - 0.2q + 1$

$0.2q^2 + 0.4q - 3 = 0$

Applying the quadratic formula: q= -5, q=3.

Sketching the graphs of MR and MC we see that MR>MC before q=3, and less after.

So, the profit is max at q=3

P(3)=TR(3)-TC(3)=9.3-4.9=4.4 hundred dollars.

Answer: The maximum profit is _____$440_____.