Friday, November 30, 2012

- Today's lecture: Section 6.5 (Loan Amortization)

- **Office hours: today 1:30-2 in CDH 109 & 3-4 in PDL C-326**

- **HOMEWORK Sections 6.4 & 6.5 are due Tuesday night.**
  + Start reviewing for the final. Most information is on the class website, more to come next week.

http://www.math.washington.edu/~nichifor/111F12.htm

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If you owe the bank $100, that's your problem. If you owe the bank $100 million, that's the bank's problem.
(John Paul Getty)

Financial Jokes - A dollar per point
A professor was giving a big test one day to his students. Once the test was over the students all handed the tests back in. The professor noticed that one of the students had attached a $100 bill to his test with a note saying "A dollar per point." The next class the professor handed the graded tests back out. This student got back his test, his test grade, and $64 change.

Section 6.5: Loan Amortizations

When a bank makes a **loan**, it is purchasing from the borrower an **ordinary annuity** that pays a fixed interest each payment period.

The **principal** of the loan (amount borrowed) **is the present value** of an ordinary annuity, and each payment received is a payment from the annuity.

Hence, the formula that applies is:

$$\boxed{P = R\,\frac{1-(1+i)^{-n}}{i}}$$

n = # of payments

amount borrowed

each payment

i = interest rate in decimal form per payment period.

Example 1:

A loan of \$10,000 is to be amortized with 10 equal quarterly payments. If the interest rate is 6%, compounded quarterly, what is the periodic payment?

$R = ?$

$R = ?$

$P = \$10,000$

$n = 10$

1 period = 1 quarter

$i = \frac{0.06}{4} = 0.015$

$$\left[10,000 = R\,\frac{1-(1.015)^{-10}}{0.015}\right] \times 0.015$$

$$150 = R\,\underbrace{\left[1-(1.015)^{-10}\right]}_{0.1383327683} \Rightarrow R = \frac{150}{0.13833\ldots} \simeq \boxed{\$1084.34}$$

Example 2:

AdriAnne and Anna's Auto Repair wants to add a new service bay. How much can they borrow at 5%, compounded quarterly for $4\frac{1}{2}$ years, if the desired quarterly payment is \$6000?

$R = \$6000$

1 period = 1 quarter

$n = (4.5\text{ years}) \times (4\text{ qtr/yr}) = 18$ quarters

$i = \frac{0.05}{4} = 0.0125$

$$P = R\,\frac{1-(1+i)^{-n}}{i}$$

$$= 6000\,\underbrace{\frac{1-(1.0125)^{-18}}{0.0125}}$$

$$= 6000 \quad 16.02954\ldots$$

$$\simeq \boxed{\$96,177.29}$$

Unpaid Balance

Every time a loan payment of $R is made:

- part of it is to pay the periodic interest on the remaining balance
- the rest goes towards reducing the remaining balance

The **unpaid balance after k payments** is equal to the present value of an ordinary annuity for the remaining n-k payments, so:

$$\boxed{A = R\,\frac{1-(1+i)^{-(n-k)}}{i}}$$

$$10{,}000 = R\,\underbrace{\frac{1-(1.011)^{-6}}{0.011}}_{5.775611911}$$

$$R = 1731.418273$$

Ex: Let's develop an "amortization schedule" for a loan of:
$10,000 for 1.5 years at 4.4% compounded quarterly.

P=$10,000
1 period = 1 quarter
n= 1.5×4 = 6 periods
i= 0.011
R= ~ $\boxed{1731.42}$

i x (current balance)

0.011 x 10,000 = 110

0.011 x 8378.58

Period	Payment	Interest	Balance reduction (Payment - Interest)	Remaining Unpaid Balance = Previous - Reduction
0				$10,000
1	$1731.42	$110	$1621.42	$8,378.58
2	$1731.42	$92.16	$1639.26	$6,739.32
3	1731.42			
4	1731.42		etc	
5	1731.42			
6	1731.42			

Filled-in table

Perio d	Payment	Interest	Balance reduction	Remaining Unpaid Balance
0				$10,000

Period	Payment	Interest	Balance reduction	Remaining Unpaid Balance
0				$10,000
1	$1,731.42	$110	$1,621.42	$8,378.58
2	$1,731.42	$92.16	$1,639.26	$6,739.32
3	$1,731.42	$74.13	$1,657.29	$5,082.03
4	$1,731.42	$55.9	$1,675.52	$3,406.51
5	$1,731.42	$37.47	$1,693.95	$1712.56
6	$1,731.42	$18.84	$1712.58	$0.00

More examples:

A couple who borrow \$90,000 for 30 years at 7.2%, compounded monthly, must make monthly payments of \$610.91.
(a) Find their unpaid balance after 1 year.
(b) During that first year, how much interest do they pay?

$P = \$90{,}000$
$n = 30 \times 12 = 360$ payments (periods)
$i = \frac{0.072}{12} = 0.006$
$R = \$610.91$

a) unpaid balance $A = ?$
after 1 year $\Rightarrow k = 12$ payments
$A =$ present value of this annuity for the remaining $\boxed{n-k} = 360 - 12 = \boxed{348}$ payments

$$A = 610.91 \underbrace{\frac{1 - (1.006)^{-348}}{0.006}}$$

$$\left(A = R \, \frac{1 - (1+i)^{-(n-k)}}{i} \right)$$

$= 610.91 \times 145.8815982$
$\approx \boxed{\$89{,}120.53}$

b) How much did they pay, total, during the first year?

$R \times 12 \text{ payments} = (\$610.91) \times (12) = \$7330.92$

How much of this went towards paying off the loan?

Initial balance $= \$90{,}000$
After 1 yr. balance $= \$89{,}120.53$

Balance reduction $= \$879.47$

[illegible] $\boxed{[illegible]}$

Balance reduction = $\$879.47$

So they paid: $\$7330.92 - \$879.47 = \boxed{\$6451.45}$

in interest
the first year