Wednesday, November 28, 2012

- Today's lecture: Sections 6.3/6.4 (more Annuities)
- **Office hours: today 1:30-2 in CDH 109 & 3-4 in PDL C-326**
- **HOMEWORK Section 6.3 is due Thursday night**.

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Math problems? Call $1-800-[(10x)(13i)^2]-\left[\frac{\sin(xy)}{2.362x}\right]$.

What do organic mathematicians burn in their fireplaces? Natural Logs.

What do mathematicians call retirement? 'the aftermath'

Recall from last time:

Suppose n **equal payments of** $\$R$ **each** are made periodically, **earning** $i \times 100\%$ **each period.**

1) **Ordinary annuity**: the payments made at the END of equal time intervals.
The future value is computed right after the last payment and is equal to:

$$\boxed{S = R\,\frac{(1+i)^n - 1}{i}}$$

2) **Annuity due**: the payments are made at the BEGINNING of the time intervals.
The future value is computed one period after the last payment
(so it draws interest for one additional period compared to an ordinary annuity)
and it equals:

$$\boxed{S = R\,\frac{(1+i)^n - 1}{i}(1+i)}$$

MORE EXAMPLES:

34. For 3 years, $400 is placed in a savings account at the beginning of each 6-month period. If the account pays interest at 10%, compounded semiannually, how much will be in the account at the end of the 3 years?

$S = ?$

$R = \$400$

$i = 0.05$ $\quad \frac{10\%}{2} = 5\%$ each 6-months

$n = 3 \times 2 = 6$ periods (of $\frac{1}{2}$ year)

$$S = 400\left(\frac{(1+0.05)^6 - 1}{0.05}\right)(1.05)$$

$$= 400\,(6.80191282)(1.05)$$

$$= \cancel{\$2724.2779}\ \boxed{\$2856.80}$$

27. How much will have to be invested at the beginning of each year at 10%, compounded annually, to pay off a debt of $50,000 in 8 years?

1 period = 1 year

annuity due

10% compounded annually => $i = 0.1$

$n = 8$

$R = ??$

$S = 50{,}000$

$$S = R\,\frac{(1+i)^n - 1}{i}(1+i)$$

$$50{,}000 = R\left(\frac{(1.1)^8 - 1}{0.1}\right)(1.1) \qquad \text{Solve for } R$$

$$\frac{50{,}000}{12.57\ldots} = R \cdot \frac{\cancel{12.57947691}}{\cancel{12.57\ldots}}$$

$$\boxed{R \cong \$3974.73}$$

More than one situation:

42. Suppose a young couple deposits $1000 at the end of each quarter in an account that earns 7.6%, compounded quarterly, for a period of 8 years. After the 8 years, they start a family and find they can contribute only $200 per quarter. If they leave the money from the first 8 years in the account and continue to contribute $200 at the end of each quarter for the next $18\frac{1}{2}$ years, how much will they have in the account (to help with their child's college expenses)?

1 First 8 years:

ordinary annuity $S = R\,\dfrac{(1+i)^n - 1}{i}$

$S = ??$

$R = \$1000$ / quarter

$i = \dfrac{0.076}{4} = 0.019$

$n = 8 \text{ years} \times 4 \text{ qtrs} = 32$

$S = 1000\,\dfrac{(1.019)^{32} - 1}{0.019} = 1000(43.4898\ldots)$

$\simeq \$43{,}489.87$ at end of 8 yrs.

2 Next $18\frac{1}{2}$ years:

2 things going on:

1) $P = \$43{,}489.87$ earning interest at 7.6% compounded quarterly for $18\frac{1}{2}$ years

$\Rightarrow S_1 = P\left(1+\dfrac{r}{n}\right)^{nt}$ 4×18.5

$= 43{,}489.87\left(1+\dfrac{0.076}{4}\right)^{74}$

$= 43{,}489.87\,(1.019)^{74}$

$\simeq \boxed{\$175{,}096.61}$

2) another ordinary annuity

$R = 200$

$i = 0.019$

$n = 18\frac{1}{2} \times 4 = 74$

$S_2 = 200\,\dfrac{(1.019)^{74} - 1}{0.019}$

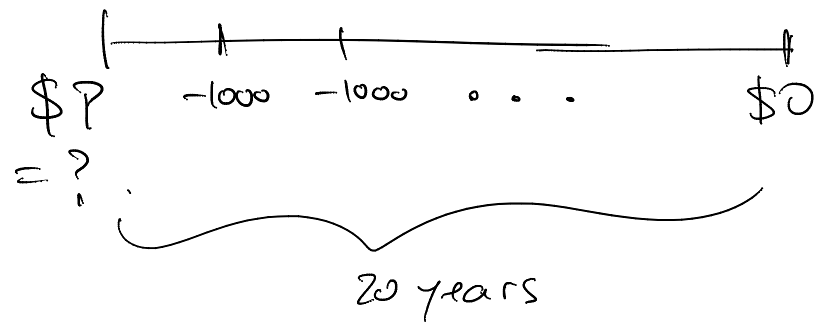
$\boxed{\simeq \$31{,}854.18}$

TOTAL: $\boxed{\$206{,}950.79}$

Section 6.4: Present value of Annuities

Motivating question:

How much is needed in an account that earns 8.4% compounded monthly in order to withdraw \$1000 at the end of each month for 20 years?



Formulas for present values of annuities:

$7\% = \boxed{0.07} \times 100\%$

If n **equal WITHDRAWALS of** $\$R$ **each** are made periodically, from an account earning $i \times 100\%$ each period.

1) **Ordinary annuity**: the withdrawals are made at the END of equal time intervals.
Then the **present value** is:

$$P = R\,\frac{1-(1+i)^{-n}}{i}$$

2) **Annuity due**: the payments are made at the BEGINNING of the time intervals.
Then the present value is:

$$P = R\,\frac{1-(1+i)^{-n}}{i}(1+i)$$

Example:

a) How much is needed in an account that earns 8.4% compounded monthly in order to withdraw $1000 at the end of each month for 20 years?

b) How about if the withdrawals are at the beginning of each month?

Ordinary annuity

$R = \$1000$ / each month

$i = \dfrac{0.084}{12} = 0.007$ /month

$n = 20 \times 12 = 240$ months

$$P = 1000\,\frac{1-(1.007)^{-240}}{0.007}$$

$$= 1000\,\underbrace{\frac{0.8125320341}{0.007}}_{116.0760049}$$

$$\simeq \boxed{\$\,116{,}076.}$$

DUE

$$P = 1000 \times \underbrace{\frac{1-(1.007)^{-240}}{0.007}}_{116{,}076} \times \boxed{1.007}$$

$$\boxed{P \simeq 116{,}888.54}$$

More examples:

7. Suppose a state lottery prize of \$5 million is to be paid in 20 payments of \$250,000 each at the end of each of the next 20 years. If money is worth 10%, compounded annually, what is the present value of the prize?

Ordinary annuity present value: $P = R\,\dfrac{1-(1+i)^{-n}}{i}$

$$= 250000\,\frac{1-(1.1)^{-20}}{0.1}$$

$$\cong \boxed{\$2,128,390.93}$$

This is how much money the lottery people need now to be able to pay you \$250,000/month for the next 20 years

If time:

36. Suppose Becky has her choice of \$10,000 at the end of each month for life or a single prize of \$1.5 million. She is 35 years old and her life expectancy is 40 more years.
    (i) Find the present value of the annuity if money is worth 7.2%, compounded monthly.
    (ii) If she takes the \$1.5 million, spends \$700,000 of it, and invests the remainder at 7.2% compounded monthly, what amount will she receive at the end of each month for the next 40 years?

ordinary annuity

exercise

$$i = \frac{0.072}{12} = 0.006$$

$$P = 10{,}000 \, \frac{1 - (1.006)^{-40 \times 12}}{0.006}$$

$$= \boxed{1{,}572{,}300.13 \, \$}$$