NAME: ________________________________ Student ID #: _________________

QUIZ SECTION:______

Math 111
Midterm II

February 16, 2006

Problem 1	17	
Problem 2	20	
Problem 3	13	
Total:	**50**	

- You may use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 5 pages in total and 3 problems. Please check your test for completeness.
- You **must show entirely how you get your answers**. Work done "in your head" cannot get credit. Correct answers with incomplete, wrong or missing work will result in little or no credit.
- Write your **final answer in the indicated units and in the indicated spaces**.
- If you need more room, use the backs of pages and indicate to the reader that you have done so. If you still need more paper, ask your TA for some more, write your name and section on it and make sure you turn it in to your TA inside your test.
- Read each question carefully. Raise your hand if you have a question.

GOOD LUCK!

Do you want me to post your grade so far next week on the class website under the last 4 digits of your SN?

☐ *Yes, please post my grade. Sign to give permission:* ________________________________

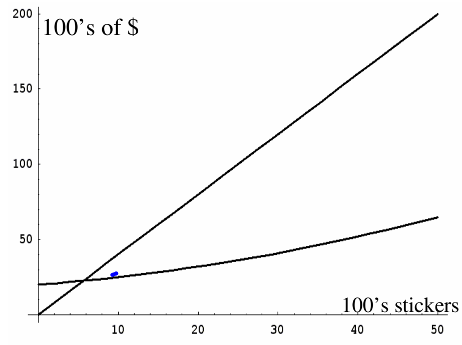
☐ *No, please don't post my grade so far.*

111B&C, Winter 2006, Midterm 2, Version 2

$\boxed{1}$(17 points) You own a business manufacturing and selling Bumper Stickers. Your total revenue and total cost, in hundreds of dollars, for a batch of q hundred Stickers are given by the following formulas:

$$TR(q) = 4q$$

$$TC(q) = 0.01q^2 + 0.4q + 10.$$



Note: 2 points were assigned for using the correct units in this problem. If you converted to the correct units some of the times, you got partial credit.

a) What is the largest number of Stickers you can produce so your total cost does not exceed $1200?

TC = 12 hundred dollars

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$$0.01q^2 + 0.4q + 10 = 12$$

$$0.01q^2 + 0.4q - 2 = 0$$

Applying the quadratic formula: q=4.4948 or -44.49.
Obviously we want the positive quantity. Convert to hundreds and round down since otherwise we exceed $1200 in TC.

Answer: No more than _____449 (or 4.49 hundreds)__ Stickers.

b) What is the maximum profit you can make? For what quantity do you get this maximal profit?

Profit(q)=TR(q)-TC(q)=(4q)-(.01 q^2+0.4q+10)=-0.01 q^2 +3.6q-10
We want the max value of this quadratic expression, so we find its vertex:

$$q = \frac{-3.6}{2(-0.01)} = 180$$

Profit(180)=-.01(180)^2+3.6(180)-10= 314

Answer: Maximum profit is _____31,400______dollars and it is obtained for ______18,000_____ Stickers.

c) What is the least number of Stickers that you must produce and sell in order to break even (make zero profit)?

We get zero profit when TR=TC, i.e. 4q=.01 q^2+0.4q+10. Get a quadratic equation and solve it using the quadratic formula. Obtain 357.2 & 2.7995. We want the least: 2.7995 hundreds of stickers, i.e. 279.95 stickers. Nearest whole number is 280.

Note: if you tried to use MR= MC, you got some nasty and unnecessary calculations. There was no reason to use MR and MC here, since you were given nice formulas for TR and TC.

Answer: ___280_______Stickers.
(round off your answer to the nearest whole number)

$\boxed{2}$ (20 points=4 each part)

You open a food stand at the HUB, selling Husky Cookies. To increase your sales, this week you run a promotion: you charge \$3 for orders of a single cookie, but for each extra cookie purchased by a customer you reduce the price per cookie by \$0.20. For example, a person ordering 3 cookies pays just \$2.60 per cookie.

Your average cost for making and selling q cookies is $AC(q) = 0.5 + \frac{3}{q}$ dollars.

a) What is your total revenue for an order of 5 cookies?

5 cookies are sold for p=3-4x0.2=\$2.2 each.
The total revenue is TR(5)=pxq=2.2x5=11

Answer: TR(5)= ___11_____ dollars.

b) Find a formula for the selling price per cookie, as a function of the quantity q ordered.

For each extra cookie, the price per cookie is reduced by \$0.2. So we have a linear price, of slope -0.2: p=-0.2q+b. Plug in q=1 and p=3 to determine the y-intercept b=3.2.

OR:

1 cookie sells for \$3, and 3 cookies sell for \$2.60. This gives two points: (1,3) & (3, 2.60) on the price versus quantity line. The slope is (2.6-3)/(3-1)=-0.2. Plug q=1 and p=3 in p=-0.2q+b to determine the y-intercept b=3.2.

Answer: The price per cookie is *p(q)* = ______*3.2-0.2q*_______.

c) For what order size do you get an Average Revenue of \$1.20?

AR(q)=TR(q)/q
TR(q)=pxq=(3.2-0.2q)q.
So AR(q)=3.2-0.2q. Set it equal to 1.2 and solve for q:

3.2-0.2q=1.2
0.2q=2
q=10

Answer: For an order size of ____10___________ cookies.

The following questions continue the previous cookie problem. Recall that your average cost for making and selling q cookies is

$$AC(q) = 0.5 + \frac{3}{q} \text{ dollars.}$$

d) What is your total cost for producing 5 cookies?

TC(q)=qAC(q)=q(0.5+3/q)=0.5q+3
TC(5)=0.5x5+3=5.5

OR:

AC(5)=0.5+3/5=1.1
TC(5)=5AC(5)=5x1.1=5.5

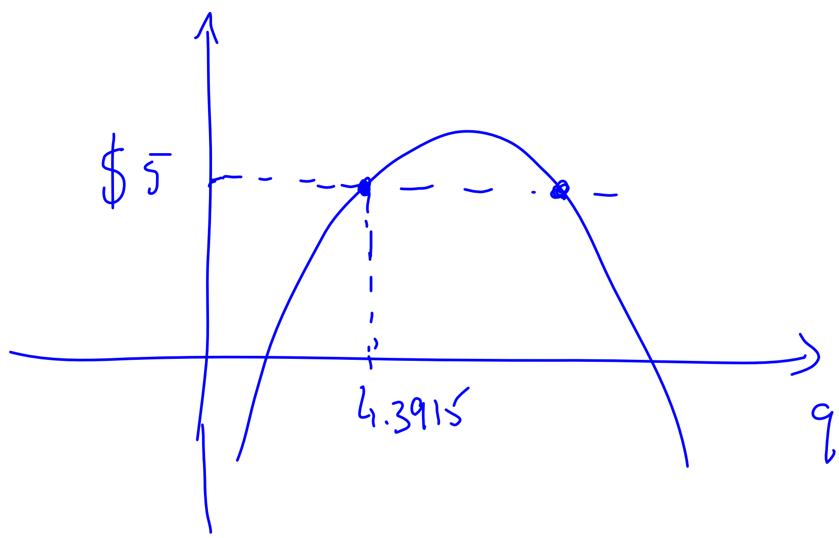
Answer: *TC(5)* =___5.5________ dollars

e) What is the smallest order size to make a profit of at least $5?

Profit(q)=TR(q)-TC(q)=$(3.2q-0.2q^2)-(0.5q+3)=-0.2q^2+2.7q-3$ (I got the TR from part c and the TC from d)

Want this to be at least $5. Set the profit equal to 5, and solve the resulting equation, using the quadratic formula. The smaller of your solutions is 4.3915. Which one is it: 4 or 5 cookies? The parabola graph of the profit opens downward, so at 4 cookies we'll make less than $5. We need to round up to 5 cookies.

rough graph:



Answer: To make a profit of $5 or more, you must produce and sell at least _____5______ cookies.

Note: Your answer should be a whole number of cookies, since you cannot produce or sell part of a cookie.

$\boxed{3}$(13 points: 4+5+4) Two cars (A and B) travel down a road for two hours.

During this two hour trip, the **distance** that car A travels from its initial position is given by

$$D_A(t) = 10t^2 + 30t\,,$$

where the time t is measured in hours, and the distance D_A is measured in miles.

a) When will car A's **average trip speed** be 47 miles per hour?

$$ATS = \frac{D_A(t)}{t} = \frac{10t^2 + 30t}{t} = 10t + 30.$$

Set equal to 47 and solve for t:

10t+30=47
10t=17
t=1.7

Answer: After ____1.7_____ hours

b) Find a formula (in terms of t) for car A's **average speed from** some time ***t*** **until 6 minutes** later. (Note that the time is measured in hours!) Simplify your formula as much as possible.

First, since the time is measured in hours, we must convert 6 min = 0.1 hrs. Then compute AS:

$$AS = \frac{D(t+0.1) - D(t)}{0.1}$$

$$\mathrm{D}(\mathrm{t}+0.1) = 10(t+0.1)^2 + 30(t+0.1) = 10t^2 + 32t + 3.1$$

$$AS = \frac{D(t+0.1) - D(t)}{0.1} = \frac{(10t^2 + 32t + 3.1) - (10t^2 + 30t)}{0.1} = \frac{2t + 3.1}{0.1} = 20t + 31$$

Answer: $AS(t) =$ __20t+31________________.

c) The second car, car B, travels with (instantaneous) **speed** $\mathrm{S_B(t) = 20t + 30}$, where the time t is still measured in hours, and the speed S_B in miles per hour. How far does car B travel during the first 6 minutes?

$S_B(0)=30$, $S_B(0.1)=20\text{x}0.1+30=32$

ATS(0.1)=(30+32)/2 =31
(midpoint of $S_B(0)=30$, $S_B(0.1)=32$, using the averaging of linear instantaneous speed from WS 17)

$D_B(0.1)$=ATS(0.1)x0.1=31x0.1=3.1

Answer: Car B travels ____3.1____ miles in the first 6 minutes.