NAME: ________________________________ Student ID #: _________________

QUIZ SECTION:______

Math 111
Midterm I SOLUTIONS

January 26, 2006

Problem 1	**11**	
Problem 2	**12**	
Problem 3	**15**	
Problem 4	**12**	
Total:	**50**	

- You are allowed to use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 5 pages in total and 4 problems. Please check your test now.
- You **must explain how you get your answers**. Correct (or incorrect) answers with no supporting work may result in little or no credit. On problems in which you use a graph, draw lines and indicate points clearly.
- Write your **final answer in the indicated spaces**. Unless otherwise noted, round your answer to two decimal digits.
- If you need more room, use the backs of pages and indicate to the reader that you have done so. If you still need more paper, ask your TA for some more, write your name and section on it and make sure you turn it in to your TA inside your test.
- Raise your hand if you have a question.

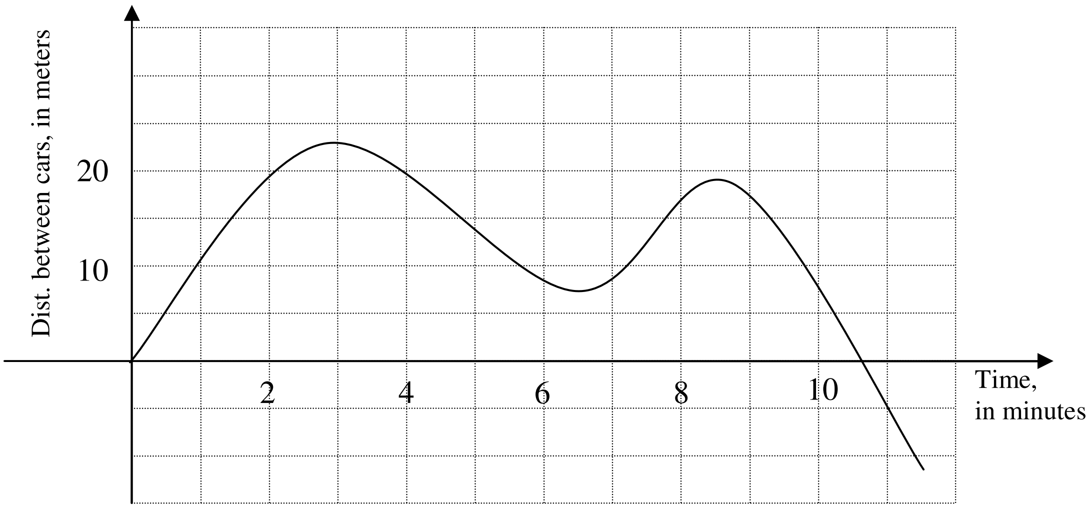
GOOD LUCK!

Do you want me to post your grade so far next week on the class website under the last 4 digits of your student ID?

☐ *Yes, please post my grade. Sign to give permission:* ________________________________

☐ *No, please don't post my grade so far.*

$\boxed{1}$(11 points: 3+4+4) Two cars, one Green and one Red, race each other starting from the same place. The graph below shows the distance **between** the two cars, from the start to the end of the race (when the graph is positive, the Green car is ahead.)



a) Who won the race? Answer: The ____Red______ car.

Explain: The graph represents the distance between the two cars throughout the race, and it's positive when the green car is ahead. Since the end of the graph is negative, the red car is ahead at the end of the race.

b) Which car traveled farthest in the 2nd minute (from t=1 to t=2)? Answer: The ____Green______ car.

Explain: Over the second minute, the distance between the green car and the red car gets larger by about 8 meters (since the graph is increasing by that much). Hence, the green car drives farther.

c) How much faster is the average speed of the Green car than the average speed of the Red car from 1 minute to 2 minutes into the race?

Answer: The Green car drives _____8_____meters per minute faster than the Red.

Work:

We're looking for the incremental rate of change of the distance between the two cars, from t=1 to t=2. This corresponds to the slope of the secant line through at the graph at t=1 and t=2. The points are approximately (1,11) and (2, 19) so

AS ≈ (19-11)/(2-1)=8 miles per minute.

2. (12 points: 3+4+5) The following table shows dollar amounts withdrawn from an ATM Cash machine over **half-hour** intervals, from noon to 5pm.

Interval starts at t=	noon	12:30	1:00	1:30	2:00	2:30	3:00	3:30	4:00	4:30
$ withdrawn	100	240	100	140	100	80	80	120	80	180

NOTE: This table shows the incremental withdrawals from the ATM. That is, from noon to 12:30, $100 were withdrawn, and from 12:30 to 1pm and additional $240.

a) How much money did the ATM dispense from 2 to 3:30 pm? Answer: ___260_____ dollars.

Work: From 2 to 3:30 there are 3 half-hour intervals, and the amounts were:
100+80+80=260

b) What is the average rate of withdrawal from noon to 2:00 pm? Answer: ____290___ dollars/hr.

Work:
We need to find the total withdrawal: 100+240+100+140=580, then divide by the time elapsed (2 hrs): 580/2=290

c) Suppose the bank deposits money into the ATM at the rate of $200 **every hour** (i.e. $200 at noon, then an additional $200 at 1 pm, etc.) What's the minimum amount of money the ATM has to contain right before noon so that it does not run out of money?

Answer: At least ______220______ dollars.

Work:
We need to find the largest negative difference between the total amount of money withdrawn from the ATM up to each time (O(t)) and the total amount of money deposited into the ATM up to each time (I(t)).

The lowest values will occur right before the deposits hit so it makes sense to look at times t right before 1pm, 2pm, etc

From noon to **just before t**	12:30	1:00	1:30	2:00	2:30	3:00	3:30	4:00	4:30	5:00
I(t) (+200 every hr)	200	200	400	400	600	600	800	800	1000	1000
O(t) (sum of $ withdrawn)	100	340	440	580	680	760	840	960	1040	1220
Surplus=I-O	100	-140	-40	-180	-80	-160	-40	-160	-40	-220

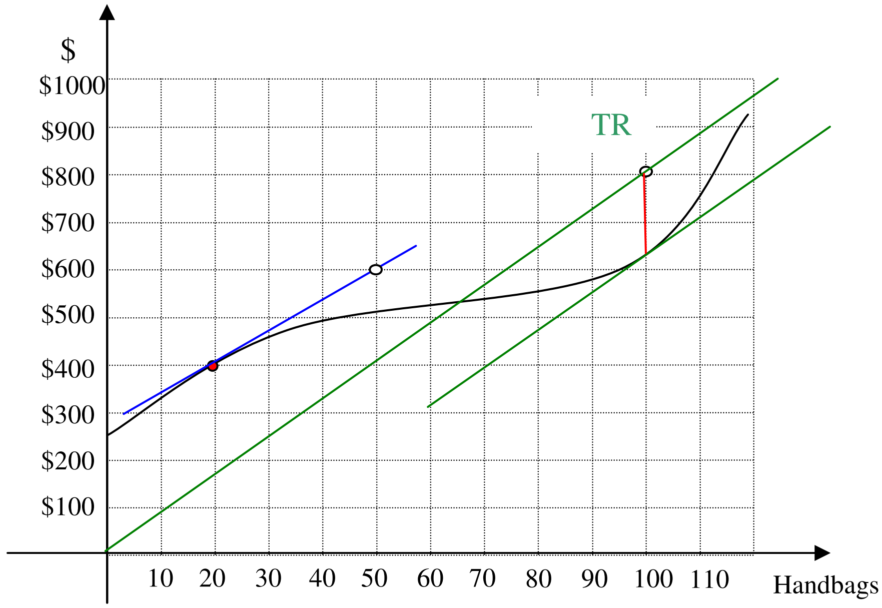
If one makes the (unsafe) extra assumption that each withdrawal happens right at the end of each interval we can look at 1pm, 2pm, etc and get the following "best case scenario" table instead:

From noon to t	12:30	1:00	1:30	2:00	2:30	3:00	3:30	4:00	4:30	5:00
I(t) (+200 every hr)	200	400	400	600	600	800	800	1000	1000	1200
O(t) (sum of $ withdrawn)	100	340	440	580	680	760	840	960	1040	1220
Surplus=I-O	100	60	-40	20	-80	40	-40	40	-40	-20

In such a case we'd get away with only $80 in the ATM at noon.

Either of these two solutions gets full credit. If you had one or the other and did not get full credit, talk to your TA.

$\boxed{3.}$(15 points: 2+3+3+3+4) You own a small business producing specialty handbags. The graph below shows your Total Cost (TC) for producing various quantities of handbags.



a) How much is your fixed cost? Answer: ____$250_____ dollars.
Explain: This is TC(0), i.e. the y-intercept of the graph.
(any value between $240 and $260 is OK)

b) What is your Marginal Cost to produce the 21st bag? Answer: ____$6.67_______dollars.
Explain:
Draw the tangent line at q=20 & compute its slope.
(600-400)/(50-20)=200/300≈6.67
(a range of values close to 6.67 were accepted, as long as the method was correct and the line was drawn carefully)

c) Suppose the current market price is $8 per bag. How many bags should you produce to earn the most profit?

Answer: ____100____ handbags.

Explain:
Since the market price p= $8 is always the same, regardless of quantity,
TR is a diagonal line of slope =p=8. Draw this line on the graph (green), then find the max distance between TR and TC, or find where a line parallel to TR becomes tangent to TC (shorter green line).
This corresponds to about q=100 on the horiz axis.

(a range of values close to 100 were accepted, as long as the method was correct, explained well, and the lines were drawn carefully)

These questions refer to the same graph above:

d) Suppose you produce 40 handbags.

What is your Average Cost per bag? Answer:___12.25______ dollars.

Work:

AC(40)=TC(40)/40≈490/40=12.25

(again, different values were accepted for the answer, as long as the work was shown and was correct, and the approx value of TC at 80 was close to 490)

e) Suppose the current market price is \$6 per handbag.

Can you earn any profit? Circle one: yes/$\boxed{\text{no}}$ and explain.

The TR line with slope 6 is always below the TC graph, so no profit is made for any quantity.

At this market price, should you produce any handbags whatsoever? Circle one: $\boxed{\text{yes}}$/no and explain.

\$6>SDP, so we can recover some of the fixed costs if we produce the right number of bags. We can see this by drawing the TR line of slope 6 and seeing that the distance between it and TC eventually gets smaller than the fixed cost. Or, we can compute the SDP and get \$6>SDP.

$\boxed{4.}$ (12 points:4+4+4) Let D(t) denote the distance (in miles) traveled by a bicyclist after t hours.

a) Translate into English: "Find a *t* such that $\frac{D(t)}{t} = 8$".

Translation: When is the average trip speed of the bicyclist 8mph?

b) Translate into functional notation: "The average trip speed of the bicyclist after 2 hours is less than his average speed from 1 to 3 hours.

Translation: $\frac{D(2)}{2} < \frac{D(3) - D(1)}{3 - 1}$

c) Suppose that after *t* hours (between 0 and 5) the distance traveled is given by the quadratic expression $D(t) = -t^2 + 10t$ miles. Use this expression to determine at what time during these five hours did the bicyclist travel 4 miles?

Answer: after _0.42____ hours.

Work:

Want t=? for D(t)=4.

Set $-t^2 + 10t = 4$, and solve for t (as in the prologue exercises)

That is, we need to find the roots of the quadratic $t^2 - 10t + 4 = 0$

$t = \frac{10 \pm \sqrt{10^2 - 4 \cdot 4}}{2} \approx 0.42 \quad or \quad 9.58$ hours. Pick the one less that 5.