Homework Set 3

Due: Monday July 18th

Section 15.10

6: Find the Jacobian of the transformation:

\[ x = 6v + 6w^2, y = 4w + 4u^2, z = 2u + 2v^2. \]

24: Evaluate the integral by making an appropriate change of variables.

\[ \iint_R 5(x + y)e^{x^2 - y^2} dA, \]

where \( R \) is the rectangle enclosed by the lines \( x - y = 0, x - y = 7, x + y = 0, \) and \( x + y = 9. \) (answer: \( \frac{5}{11}(e^{63} - 64) \))

26: Evaluate the integral by making an appropriate change of variables: \( \iint_R 6\sin(9x^2 + 4y^2) dA, \) where \( R \) is the region in the first quadrant bounded by the ellipse \( 9x^2 + 4y^2 = 1. \) (answer: \( \frac{1}{4}\pi(1 - \cos(1)) \))

14.5

1: Use the chain rule to find \( \frac{dz}{dt}, \) where \( z = x^2 + y^2 + xy, x = \sin(t), y = 8e^t. \)

13: If \( z = f(x, y), \) where \( f \) is differentiable, and \( x = g(t), g(6) = -7, g'(6) = -8, y = h(t), h(6) = 7, h'(6) = -2, f_x(-7, 7) = 1, f_y(-7, 7) = -9 \) find \( \frac{dz}{dt} \) when \( t = 6. \)

38: The radius of a right circular cone (this means that its axis is perpendicular to its base) is increasing at a rate of 1.7 \( \text{in/s} \) while its height is decreasing at a rate of 2.2 \( \text{in/s}. \) At what rate is the volume of the cone changing when the radius is 150 \( \text{in}. \) and the height is 125 \( \text{in}. \)? (answer: \( 4750\pi \text{in}^3/\text{s} \))

49: Show that any function of the form \( z = f(x + at) + g(x - at) \) is a solution to the wave equation \( \frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2} \) (that is, show that any \( z \) of this form satisfies the equation).

Hint: Let \( u = x + at, v = x - at. \)
14.6

8: Consider the following: The function \( f(x, y) = \frac{y^3}{x} \), the point \( P(1, 2) \), and the vector \( \vec{u} = \frac{1}{3}(2\vec{i} + \sqrt{5}\vec{j}) \).

Find

(a) Find the gradient of \( f \).

(b) Evaluate the gradient at the point \( P \).

(c) Find the rate of change of \( f \) at \( P \) in the direction of the vector \( \vec{u} \).

34: Suppose you are climbing a hill whose shape is given by the equation \( z = 1000 - 0.005x^2 - 0.01y^2 \), where \( x \), \( y \) and \( z \) are measured in meters, and you are standing at a point with coordinates \((60, 40, 966)\). The positive \( x \)-axis points east and the positive \( y \)-axis points north.

(a) If you walk due south, will you start to ascend or descend?

(b) If you walk northwest, will you start to ascend or descend?

(c) In which direction is the slope largest? What is the rate of ascent in that direction? At what angle above the horizontal does the path in that direction begin?

Exercise I: Find the maximum rate of change of \( f \) at the given point and the direction in which it occurs.

(a) \( f(x, y) = 12x^3 + \cos(2x + 4) \), \((2, 3)\)

(b) \( f(x, y, z) = \tan(9x + 2y + 3z) \), \((1, -9, 3)\)