Practice Test for the Final Exam

The Final will take place December 10th, 2:30pm - 4:30pm in Tho. You do not need to bring paper to write on, because there will be enough room on the test paper.

Be reminded that anything you write on the final test paper must be your own work. If there is evidence that you are claiming credit for work that is not your own work during the test period, I need to give you a zero on the test and turn the evidence over to the Dean’s Committee on Academic Conduct.

Study suggestions: To be successful on the test I highly recommend to catch up on the homework. After that repeat solving problems. Find out with which kind of problems you are still struggling. Study those over and over again, so that you develop a routine. If there are questions that you just cannot find an answer for, do not hesitate to visit me during office hours. I am more than happy to explain whatever is unclear.

Exam advise: Start with whatever problem seems easy to you. Having solved something right away calms down and gives a secure feeling. Do not waste too much time on a specific problem if you get stuck. Rather start with another problem and come back later, if there is time left. I appreciate very much, if your work is laid down in a logic order and in readable writing. If you need more space than is available on the paper, give clear instructions about where to find the rest of your work. Place a box around your final answer.

On the following pages you find review problems. You need to turn your answers in at latest by Monday, 12/01, in order to get points towards the Final. Be also reminded that you only get bonus points in the final exam if you evaluate this course online. You should get an invitation for the evaluation via email.

For TRUE/FALSE problems, you just need to circle the correct answer, unless you are asked to give a proof/counterexample.
1. True or False: Only square matrices can have an inverse.

2. True or False: If the columns space of an \((n, n)\)-matrix has dimension \(n\), then it is invertible.

3. True or False: If \(A\) is a \((5, 3)\)-matrix, then \(\text{null}(A)\) is a subspace of \(\mathbb{R}^5\).

4. True or False: If \(T : \mathbb{R}^3 \rightarrow \mathbb{R}^9\) is a homomorphism, then \(\text{range}(T)\) is a subspace of \(\mathbb{R}^9\).

5. True or False: Every matrix has a determinant.

6. True or False: If 0 is an eigenvalue of a matrix \(A\), then \(\text{nullity}(A) > 0\).

7. Let \(A\) be a \((9, 5)\)-matrix in echelon form. If \(A\) has one pivot column, what is the nullity? If \(A\) has two pivot columns, what is \(\text{rank}(A)\)?

8. Suppose that \(A\) is a \((5, 13)\)-matrix. What is the maximum possible value for the rank of \(A\), and what is the minimum possible value for the nullity of \(A\).

9. Let \(T(x) = Ax\), where

\[
A = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 7 & -6 \\ 2 & -3 & 0 \end{bmatrix}.
\]

Find the inverse function of \(T\) if it exists.

10. Check whether or not \(u\) is in \(\ker(T)\) and \(v\) is in \(\text{range}(T)\) where \(T(x) = Ax\) with

\[
A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 4 & 0 \end{bmatrix}, \quad u = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \quad v = \begin{bmatrix} 5 \\ 2 \end{bmatrix}.
\]

11. Find a basis and give the dimension for

\[
S = \text{null}(\begin{bmatrix} 2 & 3 & -7 & -1 & 4 \\ 1 & 5 & 7 & 3 & -5 \\ -1 & -1 & 5 & 1 & -3 \end{bmatrix}),
\]
which contains the vector

\[ u = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \]

12. Find \( h \), such that the following matrix has rank 2:

\[ A = \begin{bmatrix} 1 & h & -1 \\ 3 & -1 & 0 \\ -4 & 1 & 3 \end{bmatrix}. \]

13. Find the determinant of

\[ A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -3 & 2 & 2 & 1 \\ 1 & -5 & 2 & 3 \end{bmatrix}. \]

Is the matrix invertible?

14. Find the eigenvalues of

\[ A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}. \]

15. Find a basis for the eigenspace of \( A \) associated to the given eigenvalue, where

\[ A = \begin{bmatrix} 2 & 0 & 0 \\ 9 & -1 & 3 \\ 6 & -2 & 4 \end{bmatrix}, \lambda = 2. \]

16. Given \( x \) in terms of the standard basis, find the coordinate vector \( x \) with respect to \( \mathcal{B} \), where \( x = \begin{bmatrix} -2 \\ -8 \end{bmatrix}, \mathcal{B} = \{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \}. \)