Practice Test for the Midterm Exam

The Midterm will take place October 31st, during lecture time. It will cover the material of Chapter 1, Chapter 2 and Chapter 3, Section 3.1. You do not need to bring paper to write on, because there will be enough room on the test paper.

Be reminded that anything you write on the midterm test paper must be your own work. If there is evidence that you are claiming credit for work that is not your own work during the test period, I need to give you a zero on the test and turn the evidence over to the Dean’s Committee on Academic Conduct.

Study suggestions: To be successful on the test I highly recommend to catch up on the homework. After that repeat solving problems. Find out with which kind of problems you are still struggling. Study those over and over again, so that you develop a routine. If there are questions that you just cannot find an answer for, do not hesitate to visit me during office hours. I am more than happy to explain whatever is unclear.

Exam advise: Start with whatever problem seems easy to you. Having solved something right away calms down and gives a secure feeling. Do not waste too much time on a specific problem if you get stuck. Rather start with another problem and come back later, if there is time left. I appreciate very much, if your work is laid down in a logic order and in readable writing. If you need more space than is available on the paper, give clear instructions about where to find the rest of your work. Place a box around your final answer.

On the following pages you find review problems. You need to turn your answers in at latest by Monday, 10/27, in order to get points towards the Midterm.

The following set is a set of vectors which might help you with TRUE/FALSE problems, for instance. You can combine vectors or take specific vectors from this set to get a picture for yourself how a certain statement can look like in
a specific example. Otherwise you do not need to work with the set.

\[
\begin{bmatrix}
0 \\
1 \\
0 \\
0 \\
1 \\
0 \\
0 \\
0 \\
1 \\
\end{bmatrix},
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
1 \\
0 \\
1 \\
0 \\
2 \\
1 \\
\end{bmatrix}
\}
\]

For TRUE/FALSE problems, you just need to circle the correct answer, unless you are asked to give a proof/counterexample.

1. Give the definition of \textit{linear independence of vectors} \( \{u_1, u_2, \ldots, u_m\} \subseteq \mathbb{R}^n \).

2. Proof the following. If \( u \) and \( v \) are linearly independent, then so are \( u + v \) and \( u - v \).

3. Proof the following. If there are six \textit{linearly dependent} vectors in \( \mathbb{R}^6 \), then these vectors do not span \( \mathbb{R}^6 \). (Remark: This is not easy!)

4. Determine, if the following set of vectors are linearly independent and justify your answers:
(a)\[\{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \}.\]

(b)\[\{ \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 4 \end{bmatrix} \}.\]

(c) TRUE or FALSE: If \(\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}\) is linearly dependent, so is \(\{\mathbf{u}_1, \mathbf{u}_2\}\).

(d) TRUE or FALSE: If \(\mathbf{u}_4\) is not a linear combination of \(\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}\), then \(\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}\) is linearly independent.

(e) TRUE or FALSE: \(\{\mathbf{u}\}\) is always linearly independent.

(f) Give an example of a 4-set of distinct vectors \(\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\} \subset \mathbb{R}^3\) that does not span \(\mathbb{R}^3\).
5. Determine, if \[ \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix} \right\} = \mathbb{R}^3 \] and justify your answer.

6. (a) Write down an example of a homogeneous linear system with three rows and three variables.

(b) Write down the corresponding augmented matrix.

(c) If the columns of the augmented matrix of a homogeneous system are linearly independent, then the system has \( \) solution(s). If the columns are linearly dependent, then the system has \( \) solution(s).

(d) TRUE or FALSE: A homogeneous linear system can not have no solution.
(e) TRUE or FALSE: If the columns of a homogeneous system are linearly dependent, then there are infinitely many solutions to the system.

7. Solve the following linear system.

\[-4x_1 + 5x_2 - 10x_3 = 4\]
\[x_1 - 2x_2 + 3x_3 = -1\]
\[7x_1 - 17x_2 + 34x_3 = -16\]

8. Consider the function \( T : \mathbb{R}^1 \rightarrow \mathbb{R}^1, x \mapsto mx + b \). Justify your answers for the following questions.

(a) Find the values for \( m \) and \( b \), such that \( T \) is a homomorphism.

(b) Find the values for \( m \) and \( b \), such that \( T \) is an injective homomorphism.