Your Name

Your Signature

Student ID #

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- No books and no notes are allowed. You may use a calculator.
- Place a box around your final answer to each question.
- If you need more room, use the back of each page and indicate to the grader how to find the logic order of your answer.
- Raise your hand if you have questions or need more paper.
- For TRUE/FALSE problems, you just need to circle the correct answer, unless you are asked to give a proof/counterexample.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!
1. (4+5+2+2+2+2+2+2+4+5=30 points)

(a) (4 points) Give the definition of linear independence of vectors \( \{u_1, u_2, \ldots, u_m\} \subseteq \mathbb{R}^n \).

(b) (5 points) Prove the following. If \( u \) and \( v \) are linearly independent, then so are \( u + v \) and \( v \).

(c) (2 points) TRUE or FALSE: If \( u_1, u_2 \) are linearly dependent, then there is a scalar \( r \in \mathbb{R} \) such that \( u_1 = ru_2 \).

(d) (2 points) TRUE or FALSE: If \( \{u_1, u_2, u_3\} \) is a linearly dependent set, then there is a scalar \( c \) such that either \( u_1 = cu_2 \) or \( u_2 = cu_3 \) or \( u_1 = cu_3 \).

(e) (2 points) TRUE or FALSE: If \( A \) is an \((n, n)\)-matrix, then \( Ax = 0_n \) has exactly one solution.

(f) (2 points) TRUE or FALSE: If \( A \) is an \((m, n)\)-matrix with \( m < n \), then \( Ax = 0_m \) has more than one solution.

(g) (2 points) TRUE or FALSE: There are three vectors in \( \mathbb{R}^3 \) that span \( \mathbb{R}^3 \) but that are not linearly independent.

(h) (2 points) TRUE or FALSE: If \( u_1, u_2 \) are linearly independent vectors, and \( u_2, u_3 \) are linearly independent vectors, then \( u_1, u_3 \) are also linearly independent vectors.
(i) (4 points) What are the possible numbers of solutions a linear system with 308 equations and 309 variables can have? Justify your answer(s).

(j) (5 points) Determine, if the following set of vectors are linearly independent and justify your answer:

\[ A := \{ \begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 4 \end{bmatrix} \}. \]

2. (2+2+1+5+10=20 points)

(a) (2 points) TRUE or FALSE: If \( A := \{u_1, u_2, u_3\} \subseteq \mathbb{R}^3 \) is a linearly independent set and \( u_4 \in \mathbb{R}^3 \) is an arbitrary vector, then \( u_4 \) is in the span of \( A \).

(b) (2 points) TRUE or FALSE: If \( B \) is an \((m, n)\)-matrix with columns that span \( \mathbb{R}^m \), then \( Bx = b \) has a solution for all \( b \in \mathbb{R}^m \).

(c) (1 point) TRUE or FALSE: If \( \{u_1, u_2, u_3, u_4\} \) does not span \( \mathbb{R}^n \), then neither does \( \{u_1, u_2, u_3\} \).
(d) (5 points) Determine the values for $k$ such that the following vectors span $\mathbb{R}^3$. Show your work.

\[
\begin{bmatrix}
1 \\
1 \\
0 \\
\end{bmatrix}, \begin{bmatrix}
2 \\
1 \\
-2 \\
\end{bmatrix}, \begin{bmatrix}
0 \\
-1 \\
k \\
\end{bmatrix}.
\]

(e) (10 points) Circle the sets of vectors, that can never span $\mathbb{R}^3$:

- $\{u_1\}$
- $\{v_1, v_2, rv_2\}$, where $r \in \mathbb{R}$ is a scalar.
- $\{w_1, w_2, w_3\}$.
- $\{y_1, y_1 + y_2, y_1 + y_2 + y_3\}$
- $\{z_1 + z_2 + z_3\}$
(a) (5 points) Provide a matrix $A$ such that $A\mathbf{x} = T(\mathbf{x})$ for all $\mathbf{x} \in \mathbb{R}^3$.

(b) (5 points) Is $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \\ 30 \end{bmatrix}$ in the range of $T$? Show your work.

(c) (3 points) Is $S : \mathbb{R}^1 \to \mathbb{R}^2, x \mapsto \begin{bmatrix} 3x \\ x^2 \end{bmatrix}$ a linear transformation? Justify your answer.