U-Substitution for Integrals

Practice solving integrals using basic formulas and u-substitution. Each set focuses on one type of integral.

Essential Integration Formulas:

- Power Rule: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (where $n \neq -1$)
- Exponential Rule: $\int e^x dx = e^x + C$
- Constant Multiple: $\int af(x)dx = a \int f(x)dx$
- Sum Rule: $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
- U-Substitution: If $u = g(x)$, then $\int f(g(x))g'(x)dx = \int f(u)du$
- Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$ where $F'(x) = f(x)$

1 Set 1: Basic Integration - Power Rule and Exponentials

Example

Solve $\int (3x^4 + 2e^x)dx$
Solution: Step 1: Use the sum rule to split the integral into two parts:
$\int (3x^4 + 2e^x)dx = \int 3x^4 dx + \int 2e^x dx$
Step 2: Use the constant multiple rule to factor out constants:
$= 3 \int x^4 dx + 2 \int e^x dx$
Step 3: Apply the power rule to $\int x^4 dx$ (increase exponent by 1, divide by new exponent):
$\int x^4 dx = \frac{x^{4+1}}{4+1} = \frac{x^5}{5}$
Step 4: Apply the exponential rule to $\int e^x dx$ (derivative of e^x is e^x, so integral is e^x):
$\int e^x dx = e^x$
Step 5: Combine everything and add the constant of integration:
$= 3 \cdot \frac{x^5}{5} + 2 \cdot e^x + C = \frac{3x^5}{5} + 2e^x + C$

1. $\int (5x^3 + 4e^x)dx$

2. $\int (2x^6 - 3e^x)dx$

3. $\int(7x^2 + e^x + 6)dx$

4. $\int(4x^5 - 2x^3 + 5e^x)dx$

5. $\int(3x + 8e^x - x^4)dx$

2 Set 2: U-Substitution

Example

Solve $\int(2x+1)^5dx$ using $u = 2x + 1$
Solution: Step 1: Set up the substitution by defining u:
Let $u = 2x + 1$
Step 2: Find du by taking the derivative of both sides:
$du = \frac{d}{dx}(2x + 1)$, so $du = 2dx$
Step 3: Solve for dx in terms of du:
$dx = \frac{1}{2}du$
Step 4: Substitute everything into the original integral (replace $(2x+1)$ with u and dx with $\frac{1}{2}du$):
$\int(2x+1)^5dx = \int u^5 \cdot \frac{1}{2}du$
Step 5: Factor out the constant and integrate using the power rule:
$= \frac{1}{2}\int u^5du = \frac{1}{2} \cdot \frac{u^{5+1}}{5+1} = \frac{1}{2} \cdot \frac{u^6}{6} = \frac{u^6}{12}$
Step 6: Substitute back to get the answer in terms of x (replace u with $2x + 1$):
$= \frac{(2x+1)^6}{12} + C$

6. $\int (3x-2)^4 dx$ using $u = 3x - 2$

$u =$ ________________ $du =$ ________________ dx

7. $\int e^{5x} dx$ using $u = 5x$

$u =$ ________________ $du =$ ________________ dx

8. $\int (x+4)^7 dx$ using $u = x + 4$

$u =$ ________________ $du =$ ________________ dx

9. $\int (2x+3)^3 dx$ using $u = 2x + 3$

$u =$ ________________ $du =$ ________________ dx

10. $\int e^{3x+1}dx$ using $u = 3x + 1$

$u =$ ________________ $du =$ ________________ dx

3 Set 3: Fundamental Theorem of Calculus with Function Bounds

Example

Evaluate $\int_{x^2}^{x^3} tdt$
Solution: Step 1: Find the antiderivative of the integrand t:
$\int tdt = \int t^1dt = \frac{t^{1+1}}{1+1} = \frac{t^2}{2}$ (using the power rule)
Step 2: Apply the Fundamental Theorem of Calculus by evaluating the antiderivative at the upper bound minus the antiderivative at the lower bound:
$\int_{x^2}^{x^3} tdt = \frac{t^2}{2}\Big|_{x^2}^{x^3}$
Step 3: "Apply the bounds" means substitute the upper bound (x^3) for t, then subtract the result when you substitute the lower bound (x^2) for t:
$= \frac{(x^3)^2}{2} - \frac{(x^2)^2}{2}$
Step 4: Simplify using exponent rules $(x^a)^b = x^{ab}$:
$= \frac{x^6}{2} - \frac{x^4}{2}$

11. $\int_x^{x^2} t^3dt$

12. $\int_1^x t^2dt$

13. $\int_{x^2}^{2x} t dt$

14. $\int_0^{x^3} t^4 dt$

15. $\int_x^{x^4} t^5 dt$