

3D Shapes

Practice setting up equations for 3D geometric relationships. Each set focuses on one type of formula.

Essential Volume Formulas:

- Sphere: $V = \frac{4}{3}\pi r^3$, Surface Area: $A = 4\pi r^2$
- Cone: $V = \frac{1}{3}\pi r^2 h$
- Cylinder: $V = \pi r^2 h$
- Rectangular Box: $V = lwh$

1 Set 1: Spherical Volume and Surface Area

Example

A spherical balloon has radius $r = 3$ feet. Find its volume and surface area.

Solution: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 36\pi$ cubic feet
 $A = 4\pi r^2 = 4\pi(3)^2 = 36\pi$ square feet

1. A spherical water tank has radius $r = 5$ meters. Write expressions for its volume and surface area.

2. A soap bubble has radius $r = 2.5$ cm. Find its volume and surface area.

3. If a sphere has volume $V = 288\pi$ cubic inches, write the equation to find its radius. (Do not solve.)

2 Set 2: Conical Tanks with Similar Triangles

Example

A conical tank (vertex down) has height 12 ft and top radius 8 ft. Water has depth $h = 6$ ft. Find the radius of the water surface and the volume of water.

Solution: By similar triangles: $\frac{r}{h} = \frac{8}{12} = \frac{2}{3}$, so $r = \frac{2h}{3} = \frac{2(6)}{3} = 4$ ft
 $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2(6) = 32\pi$ cubic feet

4. A conical funnel has height 15 cm and top radius 9 cm. Water has depth $h = 5$ cm. Find the radius of the water surface and the volume of water.

5. An inverted cone (vertex down) has height 20 feet and top radius 6 feet. If water depth is h feet, write expressions for the water's surface radius and volume in terms of h .

6. A conical container has height 18 inches and top radius 12 inches. Water fills it to depth $h = 9$ inches. Set up the calculation for the volume of water.

3 Set 3: Cylindrical Volume

Example

A cylindrical tank has radius 4 feet and water depth 7 feet. Find the volume of water.

Solution: $V = \pi r^2 h = \pi(4)^2(7) = 112\pi$ cubic feet

7. A cylindrical container has radius 3 meters and water height 5 meters. Find the volume of water.

8. A cylinder has radius $r = 6$ inches and height $h = 10$ inches. Write the expression for its volume.

9. A cylindrical tank has radius 2.5 feet. If it contains 50 cubic feet of liquid, write the equation to find the liquid depth h .

4 Set 4: Rectangular Boxes with Constraints

Example

A box has square base with side $s = 4$ ft and height $h = 6$ ft. Find its volume.

Solution: $V = s^2h = (4)^2(6) = 96$ cubic feet

10. A rectangular box has square base with side length $s = 7$ inches and height $h = 12$ inches. Calculate its volume.

11. A box has square base with side s and height that is always twice the base length. Write the volume in terms of s only.

12. A rectangular container has length 8 ft, width 6 ft, and height h ft. If its volume is 240 cubic feet, write the equation to find h .

5 Set 5: Trapezoidal Prism Tanks

Example

A trapezoidal tank has a cross-section with parallel sides 6 ft (top) and 2 ft (bottom), and depth 4 ft. The tank is 10 ft long. Water fills to depth $h = 2$ ft. Find the volume of water.

Solution: At depth $h = 2$ ft, the top width varies linearly from 2 to 6 over 4 ft depth.

Width at depth 2: $w = 2 + \frac{(6-2) \times 2}{4} = 2 + 2 = 4$ ft

Area of water cross-section: $A = \frac{1}{2}(2 + 4) \times 2 = 6$ sq ft

Volume: $V = A \times \text{length} = 6 \times 10 = 60$ cubic feet

13. A trapezoidal irrigation channel has a bottom width of 3 meters, top width of 7 meters, and depth of 2 meters. The channel is 50 meters long. If water fills to depth $h = 1.5$ meters, find the volume of water.

14. A concrete trough has trapezoidal cross-section: bottom width 4 inches, top width 12 inches, depth 6 inches. The trough is 24 inches long. Water fills to depth h inches. Write the volume of water in terms of h .

15. A drainage ditch has parallel sides of 1 foot (bottom) and 5 feet (top), with depth 3 feet. The ditch is 100 feet long. If water fills to depth $h = 2$ feet, calculate the volume of water in the ditch.