Logarithm Practice: Preparation for Logarithmic Differentiation

Master logarithm properties and algebraic manipulation. These skills are essential for implicit differentiation and future topics!

Logarithm Rules:

$$\ln(ab) = \ln(a) + \ln(b) \quad \text{(Product Rule)}$$
$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b) \quad \text{(Quotient Rule)}$$
$$\ln(a^n) = n\ln(a) \quad \text{(Power Rule)}$$
$$\ln(e^x) = x \quad \text{(ln and e cancel)}$$
$$e^{\ln(x)} = x \quad \text{(e and ln cancel)}$$

Logarithm Properties and ln/e Relationships

Use logarithm properties to expand or simplify each expression.

Example

Expand: $\ln(x^3y^2)$
Using $\ln(ab) = \ln(a) + \ln(b)$ and $\ln(a^n) = n\ln(a)$:
$\ln(x^3y^2) = \ln(x^3) + \ln(y^2) = 3\ln(x) + 2\ln(y)$

1. Expand: $\ln(x^2y)$

2. Expand: $\ln\left(\frac{x^3}{y^2}\right)$

3. Simplify: $e^{\ln(5x)}$

4. Simplify: $\ln(e^{3x+1})$

5. Expand: $\ln(\sqrt{x} \cdot y^4)$

Taking Natural Log of Both Sides

Take the natural logarithm of both sides and use log properties to simplify.

Example

Given: $y = x^2 e^{3x}$
Take ln of both sides: $\ln(y) = \ln(x^2 e^{3x})$
Use $\ln(ab) = \ln(a) + \ln(b)$: $\ln(y) = \ln(x^2) + \ln(e^{3x})$
Use $\ln(a^n) = n\ln(a)$ and $\ln(e^u) = u$: $\ln(y) = 2\ln(x) + 3x$

6. $y = x^3 e^{2x}$

7. $y = \frac{x^4}{e^x}$

8. $y = x^x$ (Hint: This is why we need logarithmic differentiation!)

9. $y = (2x+1)^3\sqrt{x}$

10. $y = \frac{x^2 e^{5x}}{(x+1)^2}$

Algebraic Manipulation: Moving Terms

Rearrange each equation to collect all terms with x on one side and all terms with y on the other side.

Example

Given: $3x + 2y - 5x + 7y = 0$
Step 1: Collect like terms: $(3x - 5x) + (2y + 7y) = 0$ Step 2: Simplify: $-2x + 9y = 0$ Step 3: Move x terms to one side: $9y = 2x$
Final answer: $9y = 2x$

11. $2x + 3y - x + 5y = 0$

12. $4xy + 2x - 3xy + y = 0$

13. $x^2 + 2xy + y^2 - 3x^2 + xy = 0$

14. $\sin(x) + y\cos(x) - 2\sin(x) + 3y\cos(x) = 0$

15. $e^x + ye^x - 2e^x + 3ye^x = 0$

Factoring and Isolating $\frac{dy}{dx}$

Factor out $\frac{dy}{dx}$ from the terms that contain it, then solve for $\frac{dy}{dx}$.

Example

Given: $2y\frac{dy}{dx} + 3x\frac{dy}{dx} = 5x + 4$
Step 1: Factor out $\frac{dy}{dx}$: $(2y + 3x)\frac{dy}{dx} = 5x + 4$
Step 2: Divide both sides by $(2y + 3x)$: $\frac{dy}{dx} = \frac{5x+4}{2y+3x}$

16. $3y\frac{dy}{dx} + x\frac{dy}{dx} = 2x + 1$

17. $y^2\frac{dy}{dx} - 4x\frac{dy}{dx} = 6y - 3x^2$

18. $\cos(y)\frac{dy}{dx} + 2x\frac{dy}{dx} = \sin(x) + y$

19. $e^y\frac{dy}{dx} + 3xy\frac{dy}{dx} = x^2 + 5$

20. $(x+1)\frac{dy}{dx} + y^3\frac{dy}{dx} = 2xy + 7$

Taking Derivatives of Logarithmic Functions

Use the chain rule and the fact that $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$ *to find each derivative.*

Example

Find $\frac{d}{dx}[\ln(3x+1)]$
This uses the chain rule: $\frac{d}{dx}[\ln(u)] = \frac{1}{u} \cdot \frac{du}{dx}$
Here, $u = 3x + 1$, so $\frac{du}{dx} = 3$
$\frac{d}{dx}[\ln(3x+1)] = \frac{1}{3x+1} \cdot 3 = \frac{3}{3x+1}$

21. $\frac{d}{dx}[\ln(2x)]$

22. $\frac{d}{dx}[\ln(x^2+1)]$

23. $\frac{d}{dx}[\ln(5x-3)]$

24. $\frac{d}{dx}[x\ln(x)]$ (Use the product rule!)

25. $\frac{d}{dx}[\ln(\sqrt{x})]$ (Hint: Rewrite using exponent rules first)