Triangles

Practice setting up equations for triangle relationships. Each set focuses on one type of formula.

Essential Triangle Formulas:

- Pythagorean Theorem: $a^2 + b^2 = c^2$
- Similar triangles: $\frac{A}{a} = \frac{B}{b} = \frac{C}{c}$
- Right triangle trig: $\tan(\theta) = \frac{opposite}{adjacent}$, $\sin(\theta) = \frac{opposite}{hypotenuse}$, $\cos(\theta) = \frac{adjacent}{hypotenuse}$
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$
- Law of Sines: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
- Area: $A = \frac{1}{2} \times base \times height$ or $A = \frac{1}{2}ab\sin(C)$

1 Set 1: Right Triangles - Pythagorean Theorem

Example

A ladder 13 feet long leans against a wall. The bottom is 5 feet from the wall. How high up the wall does the ladder reach?
Solution: Let h = height. Using $a^2 + b^2 = c^2$: $5^2 + h^2 = 13^2$
$25 + h^2 = 169$, so $h^2 = 144$ and $h = 12$ feet

1. A guy wire supporting a radio tower is 50 feet long and is anchored 30 feet from the base of the tower. How tall is the tower?

2. A baseball diamond is a square with 90-foot sides. What is the distance from home plate to second base?

3. A boat travels 12 miles east, then 16 miles north. How far is the boat from its starting point?

2 Set 2: Right Triangles - Trigonometry

Example

A kite string makes a 60° angle with the horizontal. The string is 100 feet long. How high is the kite?
Solution: Let h = height. Using $\sin(\theta) = \frac{opposite}{hypotenuse}$:
$\sin(60) = \frac{h}{100}$, so $h = 100\sin(60) = 100 \cdot \frac{\sqrt{3}}{2} = 50\sqrt{3}$ feet

4. From a lighthouse 80 feet above sea level, the angle of depression to a boat is 25°. How far is the boat from the base of the lighthouse?

5. A ramp for wheelchair access makes a 5° angle with the horizontal and rises 3 feet. How long is the ramp along its inclined surface?

6. A surveyor stands 100 feet from a building and measures the angle of elevation to the roof as 35°. How tall is the building?

3 Set 3: Similar Triangles

Example

A person 6 feet tall casts a 4-foot shadow. At the same time, a flagpole casts a 28-foot shadow. How tall is the flagpole?
Solution: Set up proportion: $\frac{\text{person height}}{\text{person shadow}} = \frac{\text{flagpole height}}{\text{flagpole shadow}}$
$\frac{6}{4} = \frac{h}{28}$, so $h = \frac{6 \times 28}{4} = 42$ feet

7. A tree casts a 15-foot shadow when a 4-foot fence post casts a 3-foot shadow. How tall is the tree?

8. On a map, 2 inches represents 50 miles. If two cities are 7 inches apart on the map, what is the actual distance between them?

9. A model airplane has a wingspan of 3 feet and represents a real plane with a wingspan of 180 feet. If the model is 2.5 feet long, how long is the real airplane?

4 Set 4: Non-Right Triangles - Law of Cosines and Law of Sines

Example

A triangle has sides $a = 8$, $b = 10$, and included angle $C = 60$. Find side c.
Solution: Using Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$
$c^2 = 8^2 + 10^2 - 2(8)(10)\cos(60) = 64 + 100 - 160(\frac{1}{2}) = 84$
So $c = \sqrt{84} = 2\sqrt{21}$

10. Two ships leave port at the same time. One travels 25 miles in direction N30°E, the other travels 40 miles due east. How far apart are the ships?

11. A triangle has sides of length 7 cm and 9 cm with an included angle of 120°. Find the length of the third side.

12. In triangle ABC, side $a = 12$, side $b = 15$, and angle $A = 40$. Use the Law of Sines to find angle B.

5 Set 5: Triangular and Trapezoidal Tank Cross-Sections

Example

A triangular trough has a cross-section that is an equilateral triangle with side length 6 feet. Water fills to depth $h = 3$ feet. Find the width of the water surface.

Solution: In an equilateral triangle with side 6, the height is $3\sqrt{3}$. By similar triangles:

$\frac{\text{water width}}{6} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$

Water width $= \frac{6}{\sqrt{3}} = 2\sqrt{3}$ feet

13. A water channel has a triangular cross-section with base 8 meters and height 5 meters. If water fills to depth $h = 2$ meters, find the width of the water surface at the top.

14. A concrete irrigation ditch has a trapezoidal cross-section: bottom width 2 feet, top width 8 feet, depth 4 feet. When water is 3 feet deep, what is the width of the water surface?

15. A storm drain has an isosceles triangular cross-section with base 10 feet and height 6 feet. Water fills to depth h feet. Write an expression for the width of the water surface in terms of h.