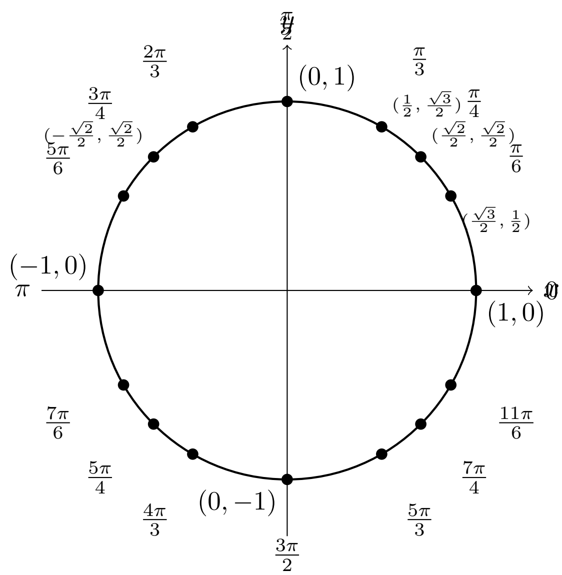
Algebra Review and Curve Sketching

Review essential algebra skills and practice curve sketching techniques. Each set focuses on different fundamental concepts.

Essential Formulas and Concepts:

- Unit Circle: $\sin(0) = 0$, $\cos(0) = 1$, $\sin(\frac{\pi}{2}) = 1$, $\cos(\frac{\pi}{2}) = 0$, etc.
- Exponential Properties: $e^{a+b} = e^a \cdot e^b$, $e^{a-b} = \frac{e^a}{e^b}$, $(e^a)^b = e^{ab}$
- Logarithm Properties: $\ln(ab) = \ln(a) + \ln(b)$, $\ln(\frac{a}{b}) = \ln(a) - \ln(b)$, $\ln(a^b) = b\ln(a)$
- $\ln(e^x) = x$ and $e^{\ln(x)} = x$
- Critical Points: Where $f'(x) = 0$ or $f'(x)$ is undefined
- Second Derivative Test: If $f''(c) > 0$, then $(c, f(c))$ is a local minimum; if $f''(c) < 0$, then $(c, f(c))$ is a local maximum

1 Set 1: Unit Circle Review



Example

Find $\sin(\frac{\pi}{4})$ and $\cos(\frac{\pi}{4})$
Solution: Step 1: Recall that $\frac{\pi}{4}$ radians equals 45°
Step 2: On the unit circle, the angle $\frac{\pi}{4}$ corresponds to the point where x and y coordinates are equal
Step 3: Since we're on the unit circle ($x^2 + y^2 = 1$) and $x = y$, we have $x^2 + x^2 = 1$
Step 4: Solve: $2x^2 = 1$, so $x^2 = \frac{1}{2}$, which gives us $x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
Step 5: Therefore, $\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ and $\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

1. Find $\sin(\frac{\pi}{6})$ and $\cos(\frac{\pi}{6})$

2. Find $\sin(\frac{\pi}{3})$ and $\cos(\frac{\pi}{3})$

3. Find $\sin(\frac{\pi}{2})$ and $\cos(\frac{\pi}{2})$

4. Find $\sin(\pi)$ and $\cos(\pi)$

5. Find $\sin(\frac{3\pi}{2})$ and $\cos(\frac{3\pi}{2})$

2 Set 2: Exponential and Logarithm Equations

Example

Solve for x: $\ln(2x-1) = 3$
Solution: Step 1: To eliminate the natural logarithm, apply the exponential function to both sides
$e^{\ln(2x-1)} = e^3$
Step 2: Use the property that $e^{\ln(a)} = a$ to simplify the left side
$2x - 1 = e^3$
Step 3: Solve for x by adding 1 to both sides
$2x = e^3 + 1$
Step 4: Divide both sides by 2
$x = \frac{e^3+1}{2}$
Step 5: Check: We need $2x - 1 > 0$ for the logarithm to be defined. Since $e^3 > 0$, we have $x > 0$, so $2x - 1 = e^3 > 0$

6. Solve for x: $e^{2x} = 8$

7. Solve for x: $\ln(x + 3) = 2$

8. Solve for x: $e^{x-1} = 5$

9. Simplify: $\ln(e^{3x})$

10. Solve for x: $\ln(2x) - \ln(x-1) = \ln(3)$

3 Set 3: Curve Sketching

Example

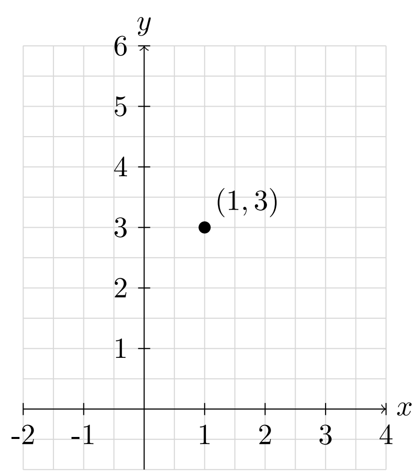
Sketch a curve that passes through the critical point $(2, -1)$ where the second derivative is negative.

Solution: Step 1: Identify what we know about the point $(2, -1)$
- It's a critical point, so $f'(2) = 0$ (the tangent line is horizontal)
- The second derivative is negative, so $f''(2) < 0$

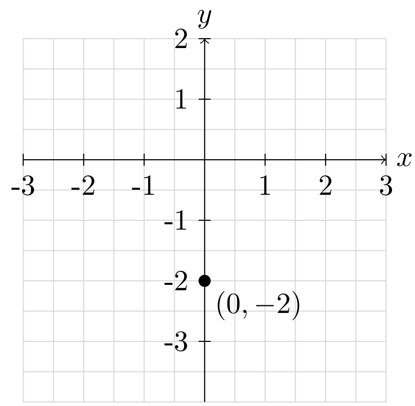
Step 2: Apply the second derivative test
Since $f''(2) < 0$, the point $(2, -1)$ is a local maximum
Step 3: Consider the behavior around this point
- The curve is concave down at $x = 2$ (since $f''(2) < 0$)
- The curve has a horizontal tangent at $(2, -1)$
- The curve rises to the left and right of $x = 2$, then falls away

Step 4: Sketch the curve
Draw a smooth curve that passes through $(2, -1)$ with a horizontal tangent line, forming a "hill" or upside-down U shape around this point.

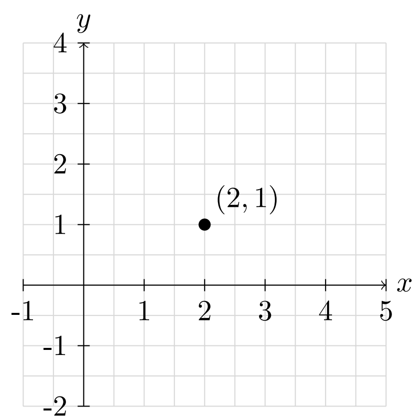
11. Sketch a curve that passes through the critical point $(1, 3)$ where the second derivative is positive.



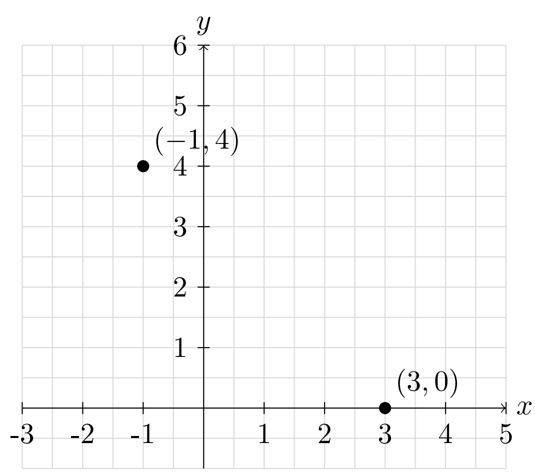
12. Sketch a curve that has a critical point at $(0, -2)$ and is concave up everywhere.



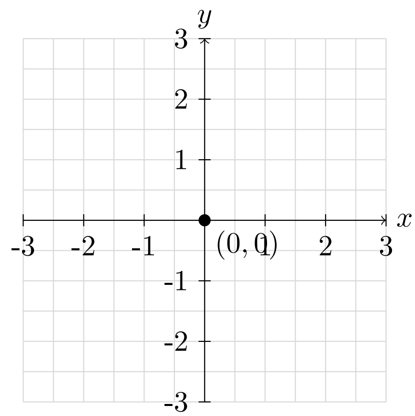
13. Sketch a curve that passes through $(2, 1)$, has a horizontal tangent line at that point, and changes from concave down to concave up at $x = 2$.



14. Sketch a curve that has a local maximum at $(-1, 4)$ and a local minimum at $(3, 0)$.



15. Sketch a curve that passes through the origin $(0, 0)$, has a critical point there, and $f''(0) = 0$ (inflection point).



y
3
2
1
x
-3
-2
-1
(0, 0)
1
2
3
-1
-2
-3